Elastic Collisions in One Dimension

Exercise A

1 In each part of this question, the two diagrams show the speeds and directions of motion of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. Find the coefficient of restitution e in each case.

	Before collision		After collision	
a	$6\,\text{m s}^{-1}$ → A	At rest B	At rest A	$4\,\text{m s}^{-1}$ → B
b	$4\,\text{m s}^{-1}$ → A	$2\,\text{m s}^{-1}$ → B	$2\,\text{m s}^{-1}$ → A	$3\,\text{m s}^{-1}$ → B
c	$9\,\text{m s}^{-1}$ → A	$6\,\text{m s}^{-1}$ ← B	$3\,\text{m s}^{-1}$ ← A	$2\,\text{m s}^{-1}$ → B

2 In each part of this question, the two diagrams show the speeds and directions of motion of two particles A and B just before a collision, and their velocities relative to the initial directon of A just after a collision. The particles move on a smooth horizontal plane. The masses of A and B and the coefficients of restitution e are also given. Find the values of v_1 and v_2 in each case.

	Before collision		After collision	
a $e = \frac{1}{2}$	$6\,\text{m s}^{-1}$ → A (0.25 kg)	At rest B (0.5 kg)	$v_1\,\text{m s}^{-1}$ → A (0.25 kg)	$v_2\,\text{m s}^{-1}$ → B (0.5 kg)
b $e = 0.25$	$4\,\text{m s}^{-1}$ → A (2 kg)	$2\,\text{m s}^{-1}$ → B (3 kg)	$v_1\,\text{m s}^{-1}$ → A (2 kg)	$v_2\,\text{m s}^{-1}$ → B (3 kg)
c $e = \frac{1}{7}$	$8\,\text{m s}^{-1}$ → A (3 kg)	$6\,\text{m s}^{-1}$ ← B (1 kg)	$v_1\,\text{m s}^{-1}$ → A (3 kg)	$v_2\,\text{m s}^{-1}$ → B (1 kg)
d $e = \frac{2}{3}$	$6\,\text{m s}^{-1}$ → A (400 g)	$6\,\text{m s}^{-1}$ ← B (400 g)	$v_1\,\text{m s}^{-1}$ → A (400 g)	$v_2\,\text{m s}^{-1}$ → B (400 g)
e $e = \frac{1}{5}$	$3\,\text{m s}^{-1}$ → A (5 kg)	$12\,\text{m s}^{-1}$ ← B (4 kg)	$v_1\,\text{m s}^{-1}$ → A (5 kg)	$v_2\,\text{m s}^{-1}$ → B (4 kg)

3 A small smooth sphere A of mass 1 kg is travelling along a straight line on a smooth horizontal plane with speed 4 m s^{-1} when it collides with a second smooth sphere B of the same radius, with mass 2 kg and travelling in the same direction as A with speed 2.5 m s^{-1}. After the collision, A continues in the same direction with speed 2 m s^{-1}. Find:

a the speed of B after the collision

b the coefficient of restitution for the spheres.

4 Two spheres A and B have masses 2 kg and 6 kg respectively. A and B move towards each other in opposite directions along the same straight line on a smooth horizontal surface with speeds 4 m s^{-1} and 6 m s^{-1} respectively. If the coefficient of restitution is $\frac{1}{5}$, find the velocities of the spheres after the collision and the magnitude of the impulse given to each sphere.

5 Two particles P and Q of masses $2m$ and $3m$ respectively are moving in opposite directions towards each other. Each particle is travelling with speed u. Given that Q is brought to rest by the collision, find the speed of P after the collision, and the coefficient of restitution between the particles.

6 Two particles A and B are travelling along the same straight line in the same direction on a smooth horizontal surface with speeds $3u$ and u respectively. Particle A catches up and collides with particle B. If the mass of B is twice that of A and the coefficient of restitution is e, find, in terms of e and u, expressions for the speeds of A and B after the collision.

7 Two identical particles of mass m are projected towards each other along the same straight line on a smooth horizontal surface with speeds of $2u$ and $3u$. After the collision, the directions of motion of both particles are reversed. Show that this implies that the coefficient of restitution e satisfies the inequality $e > \frac{1}{5}$

8 Two particles A and B of masses m and km respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed u so as to collide directly with B, which is at rest. After the collision B moves with speed $\frac{3}{10}u$.

a Find, in terms of u and the constant k, the speed of A after the collision. **(4 marks)**

b By using Newton's law of restitution, show that $\frac{7}{3} \leqslant k \leqslant \frac{17}{3}$ **(5 marks)**

Problem-solving

In part **b** use the limits of e to set up an inequality.

9 Two particles A and B of masses m and $3m$ respectively are placed on a smooth horizontal plane. Particle A is made to move on the plane with speed $2u$ so as to collide directly with B, which is moving in the same direction with speed u. After the collision B moves with speed ku, where k is a constant.

a Find, in terms of u and the constant k, the speed of A after the collision. **(4 marks)**

b By using Newton's law of restitution, show that $\frac{5}{4} \leqslant k \leqslant \frac{3}{2}$ **(5 marks)**

10 A particle P of mass m is moving with speed $4u$ on a smooth horizontal plane. The particle collides directly with a particle Q of mass $3m$ moving with speed $2u$ in the same direction as P. The coefficient of restitution between P and Q is e.

a Show that the speed of Q after the collision is $\frac{u}{2}(5 + e)$. **(6 marks)**

b Find the speed of P after the collision, giving your answer in terms of e. **(4 marks)**

c Show that the direction of motion of P is unchanged by the collision. **(2 marks)**

d Given that the magnitude of the impulse of P on Q is $2mu$, find the value of e. **(4 marks)**

Challenge

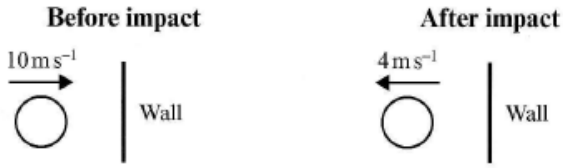
Two particles P and Q of masses $3m$ kg and m kg respectively move towards each other in a straight line in opposite directions on a smooth horizontal surface. P has initial speed 2 m s^{-1} and Q has initial speed u m s^{-1}. After P and Q collide, both particles move in the same direction as P's original motion. Given that, after the impact, Q moves with twice the speed of P and that the coefficient of restitution between P and Q is $\frac{1}{4}$, show that $u = \frac{14}{9}$

Exercise B

Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$.

1 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. In each case, find the value of the coefficient of restitution e.

a



b

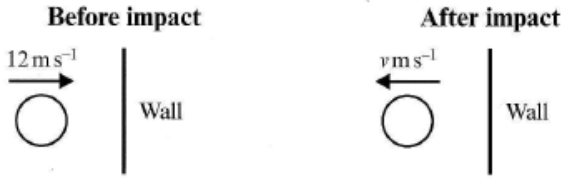


2 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. The value of e is given in each case. Find the speed of the sphere after the collision in each case.

a $e = \frac{1}{2}$

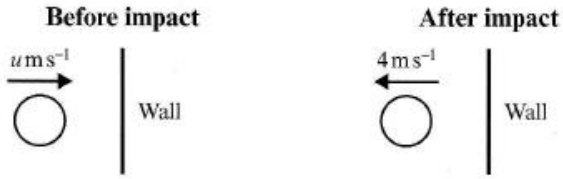


b $e = \frac{1}{4}$

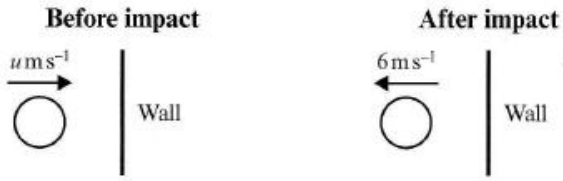


3 A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed and direction of motion of the sphere before and after the collision. The value of e is also given. Find the speed of the sphere before the collision in each case.

a $e = \frac{1}{2}$



b $e = \frac{3}{4}$



4 A small smooth sphere of mass 0.3 kg is moving on a smooth horizontal table with a speed of $10\,\text{m s}^{-1}$ when it collides normally with a fixed smooth wall. It rebounds with a speed of $7.5\,\text{m s}^{-1}$. Find the coefficient of restitution between the sphere and the wall.

5 A particle falls 2.5 m from rest onto a smooth horizontal plane. It then rebounds to a height of 1.5 m. Find the coefficient of restitution between the particle and the plane. Give your answer to 2 significant figures.

6 A particle falls 3 m from rest onto a smooth horizontal plane. It then rebounds to a height h m. The coefficient of restitution between the particle and the plane is 0.25.

a Find the value of h. **(4 marks)**

b Without further calculation, state how your answer to part **a** would change if $e > 0.25$ **(1 mark)**

7 A small smooth sphere falls from rest onto a smooth horizontal plane and rebounds from the plane. It takes 2 seconds to reach the plane then another 2 seconds to reach the plane a second time. Find the coefficient of restitution between the sphere and the plane. **(7 marks)**

8 A small smooth sphere falls from rest onto a smooth horizontal plane. It takes 3 seconds to reach the plane. The coefficient of restitution between the sphere and the plane is 0.49. Find the time it takes after rebound for the sphere to reach the plane a second time. **(7 marks)**

9 A particle falls from rest from a height of h m above level ground. After rebounding the particle reaches a maximum height of $\frac{1}{2}h$. Find, in terms of g and h, the height of the particle 1 second after it hits the ground. **(8 marks)**

Challenge

A particle P falls from rest from a height of h m above level ground. Show that after hitting the ground, the maximum height that the particle reaches is he^2, where e is the coefficient of restitution between the particle and the ground.

Exercise C

1 A particle A of mass 500 g lies at rest on a smooth horizontal table. A second particle B of mass 600 g is projected along the table with speed 6 m s^{-1} and collides directly with A. If the collision reduces the speed of B to 1 m s^{-1}, without changing its direction, find:

a the speed of A after the collision **b** the loss of kinetic energy due to the collision.

2 Two particles A and B of masses m and $2m$ respectively move towards each other in opposite directions with speeds u and $2u$. If the coefficient of restitution between the particles is $\frac{2}{3}$, find the velocities of A and of B after the collision. Find also, in terms of m and u, the loss of kinetic energy due to the collision.

3 A particle of mass 3 kg moving with speed 6 m s^{-1} collides directly with a particle of mass 5 kg moving in the opposite direction with speed 2 m s^{-1}. The particles coalesce and move with velocity v after the collision. Find the loss of kinetic energy due to the impact.

4 A billiard ball of mass 200 g strikes a smooth cushion at right angles. Its speed before the impact is 2.5 m s^{-1} and the coefficient of restitution is $\frac{4}{5}$. Find the loss in kinetic energy of the billiard ball due to the impact.

5 A bullet of mass 0.15 kg moving horizontally at 402 m s^{-1} embeds itself in a sandbag of mass 30 kg, which is suspended freely. Assuming that the sandbag is stationary before the impact, find:

a the common velocity of the bullet and the sandbag after the impact

b the loss of kinetic energy due to the impact.

6 A bullet is fired horizontally from a rifle. The rifle has mass 4.8 kg and the bullet has mass 20 g. The initial speed of the bullet is 400 m s^{-1}. Find:

a the initial speed with which the rifle recoils

b the total kinetic energy generated as a result of firing the bullet.

7 A train of mass 30 tonnes moving with a small speed V impacts upon a number of stationary carriages each weighing 6 tonnes. The complete train and carriages now move forward with a speed of $\frac{5}{8}V$. Find:

a the number of stationary carriages

b the fraction of the original kinetic energy lost in the impact.

8 A truck of mass 5 tonnes is moving in a straight line at 1.5 m s^{-1} towards a second stationary truck of mass 10 tonnes which is at rest. The trucks collide, and after the impact the second truck moves at 0.6 m s^{-1}. Modelling the trucks as particles, find:

Hint 1 tonne = 1000 kg

a the velocity of the first truck after the impact **(3 marks)**

b the coefficient of restitution between the two trucks **(2 marks)**

c the loss of kinetic energy due to the impact. **(3 marks)**

9 A particle of mass m moves in a straight line with speed v when it explodes into two parts, one of mass $\frac{1}{3}m$ and the other of mass $\frac{2}{3}m$, both moving in the same direction as before. If the explosion increases the energy of the system by $\frac{1}{4}mu^2$, where u is a positive constant, find the speeds of the particles immediately after the explosion. Give your answers in terms of u and v.

10 A small smooth sphere A of mass 2 kg moves at 4 m s^{-1} on a smooth horizontal table. It collides directly with a second small smooth sphere B of mass 3 kg, which is moving in the same direction at a speed of 1 m s^{-1}. The loss of kinetic energy due to the collision is 3 J. After the collision A and B continue to move in the same direction with speeds u and v respectively.

a Show that $5v^2 - 22v + 21 = 0$. **(6 marks)**

b Hence find u and v, carefully justifying your choice of solutions. **(5 marks)**

11 Two particles A and B, of masses 2 kg and 5 kg respectively, are connected by a light inextensible string. The particles are side by side on a smooth floor and A is projected with speed 7 m s^{-1} directly away from B. When the string becomes taut, particle B is jerked into motion and A and B then move with a common speed in the direction of the original velocity of A. Find:

a the common speed of the particles after the string becomes taut **(4 marks)**

b the loss of total kinetic energy due to the jerk. **(3 marks)**

12 Two particles A and B, of masses m and M respectively, are connected by a light inextensible string. The particles are side by side on a smooth floor and A is projected with speed u directly away from B. When the string becomes taut, particle B is jerked into motion and A and B then move with a common speed in the direction of the original projection of A.

Show that the loss of total kinetic energy due to the jerk is $\dfrac{mMu^2}{2(m+M)}$ **(8 marks)**

Problem-solving

Start by finding an expression for the common speed, v, after the string becomes taut.

13 Two particles of masses 3 kg and 5 kg lie on a smooth table and are connected by a slack inextensible string. The first particle is projected along the table with a velocity of 20 m s^{-1} directly away from the second particle. Find:

a the velocity of each particle after the string has become taut **(6 marks)**

b the difference between the kinetic energies of the system when the string is slack and when it is taut. **(3 marks)**

14 Three small spheres of masses 20 g, 40 g and 60 g respectively lie in order in a straight line on a large smooth table. The distance between adjacent spheres is 10 cm. Two slack strings, each 70 cm in length, connect the first sphere with the second, and the second sphere with the third. The 60 g sphere is projected with a speed of 5 m s^{-1}, directly away from the other two. Find:

a the time which elapses before the 20 g sphere begins to move and the speed with which it starts **(8 marks)**

b the loss in kinetic energy resulting from the two jerks. **(5 marks)**

Challenge

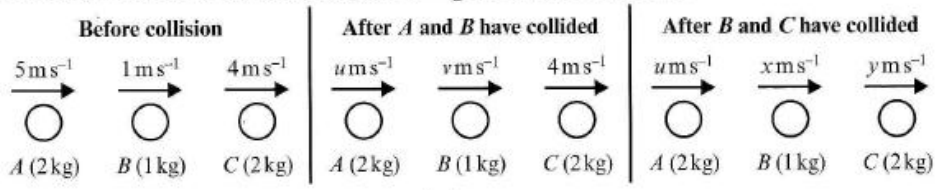
Two small spheres A and B with masses 4 kg and 1 kg respectively lie on a large smooth surface. A and B are connected by a light inextensible string and are projected directly towards each other with speeds 2 m s^{-1} and 3 m s^{-1} respectively. The coefficient of restitution for the collision of A and B is 0.8. After the collision both particles move in the direction in which A was originally moving. Find the kinetic energy of the system when the string becomes taut.

Exercise D

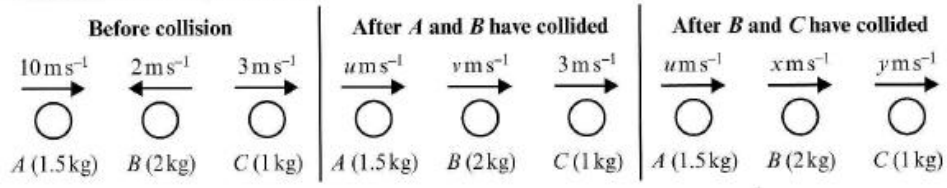
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.

1 Three small smooth spheres A, B and C move along the same straight line on a horizontal plane. Sphere A collides with sphere B and then sphere B collides with sphere C. The diagrams show the velocities before the first collision, after the first collision between A and B and then after the collision between B and C.

a Find the values of u, v, x and y, if $e = \frac{1}{2}$ for both collisions.



b Find the values of u, v, x and y, if $e = \frac{1}{6}$ for the collision between A and B and $e = \frac{1}{2}$ for the collision between B and C.



2 Three perfectly elastic particles A, B and C of masses $3m$, $5m$ and $4m$ respectively lie at rest on a straight line on a smooth horizontal table with B between A and C. Particle A is projected directly towards B with speed 6 m s^{-1} and after A has collided with B, B then collides with C. Find the velocity of each particle after the second impact.

Hint Perfectly elastic means $e = 1$.

3 Three identical smooth spheres A, B and C, each of mass m, lie at rest on a straight line on a smooth horizontal table. Sphere A is projected with speed u to strike sphere B directly. Sphere B then strikes sphere C directly. The coefficient of restitution between any two spheres is e, $e \neq 1$.

a Find the speeds in terms of u and e of the spheres after these two collisions.

b Show that A will catch up with B and there will be a further collision.

4 Three identical spheres A, B and C of equal mass m move along the same straight line on a horizontal plane. A and B are moving in opposite directions towards each other with speeds $4u$ and $2u$ respectively. C is moving in the same direction as A with speed $3u$.

a If the coefficient of restitution between any two of the spheres is e, show that B will only collide with C if $e > \frac{2}{3}$ **(8 marks)**

b Find the direction of motion of A after collision, if $e > \frac{2}{3}$ **(2 marks)**

5 Two particles P and Q of masses $2m$ and $3m$ respectively are moving in opposite directions on a smooth plane with speeds $4u$ and $2u$ respectively. The particles collide directly. The direction of motion of Q is reversed by the impact and its speed after impact is u. This particle then hits a smooth vertical wall perpendicular to its direction of motion. The coefficient of restitution between Q and the wall is $\frac{2}{3}$. In the subsequent motion, there is a further collision between Q and P. Find the velocities of P and Q after this collision. **(8 marks)**

6 Two small smooth spheres P and Q have masses m and $3m$ respectively. Sphere P is moving with speed $12u$ on a smooth horizontal table when it collides directly with Q which is at rest on the table. The coefficient of restitution between P and Q is $\frac{2}{3}$

a Find the velocities of P and Q immediately after the collision. **(6 marks)**

After the collision Q hits a smooth vertical wall perpendicular to the direction of its motion. The coefficient of restitution between Q and the wall is $\frac{4}{5}$. Q then collides with P a second time.

b Find the velocities of P and Q after the second collision between P and Q. **(8 marks)**

7 A small table tennis ball, which may be modelled as a particle, falls from rest at a height 40 cm onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is 0.7.

a Find the height to which the ball rebounds after:

i the first bounce

ii the second bounce. **(8 marks)**

b Describe the subsequent motion of the ball. **(1 mark)**

c Find the total distance travelled by the ball before it comes to rest. **(5 marks)**

Problem-solving

Find the ratio between the heights of successive bounces according to the model.

d Give one reason why your answer to part **c** is unrealistic. **(1 mark)**

8 A small smooth ball, which may be modelled as a particle, falls from rest at a height H onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is e.

a Find, in terms of H and e, the height to which the ball rebounds after the first bounce. **(9 marks)**

b Find, in terms of H and e, the height to which the ball rebounds after the second bounce. **(6 marks)**

c Find an expression for the total distance travelled by the ball before it comes to rest. **(6 marks)**

Problem-solving

Draw clear diagrams for each stage of the motion of the ball.

9 A ball B lies on a smooth horizontal plane between two smooth, parallel, vertical walls W_1 and W_2 that are d m apart. B is initially halfway between the walls and is projected with velocity $2\,\text{m s}^{-1}$ towards W_2. The coefficient of restitution between B and W_2 is e_2.

The ball then travels towards and strikes W_1. The coefficient of restitution between B and W_1 is e_1.

The ball then travels towards W_2 again.

Find, in terms of d, e_1 and e_2, the total time elapsed from the moment the ball is projected to the time when it strikes W_2 for the second time. **(9 marks)**

Challenge

Two particles P and Q, of equal mass, lie on a smooth horizontal plane between two smooth parallel, vertical walls W_1 and W_2 that are 4 m apart. P and Q are projected towards each other with speeds $2\,\text{m s}^{-1}$ and $1\,\text{m s}^{-1}$ respectively in such a way that they collide directly at a point equidistant from W_1 and W_2. The coefficient of restitution between the particles is 0.5. P then travels towards W_1, and Q travels towards W_2 and strikes W_2 perpendicularly. The coefficient of restitution between Q and W_2 is 0.4. Show that in the subsequent motion, P strikes W_1 before Q collides with P for a second time.

Exercise E

1 Two identical spheres, moving in opposite directions, collide directly. As a result of the impact one of the spheres is brought to rest. The coefficient of restitution between the spheres is $\frac{1}{3}$. Show that the ratio of the speeds of the spheres before the impact is 2 : 1.

2 A particle P of mass m is moving in a straight line with speed $\frac{1}{4}u$ at the instant when it collides directly with a particle Q, of mass λm, which is at rest. The coefficient of restitution between P and Q is $\frac{1}{4}$. Given that P comes to rest immediately after hitting Q find the value of λ.

3 A boy of mass m dives off a boat of mass M which was previously at rest. Immediately after diving off, the boy has horizontal speed v and the boat has horizontal speed V.

a Modelling the boy and the boat as particles in free space, find an expression for V in terms of v. **(4 marks)**

b Prove that the total kinetic energy of the boy and the boat is $\frac{m(m+M)v^2}{2M}$ **(4 marks)**

c Criticise the model in respect of the boat. **(1 mark)**

4 Two spheres P and Q of masses 4 kg and 2 kg respectively are travelling towards each other in opposite directions along a straight line on a smooth horizontal surface. Initially, P has a speed of $5\,\text{m s}^{-1}$ and Q has a speed of $3\,\text{m s}^{-1}$. After the collision the direction of Q is reversed and it is travelling at a speed of $2\,\text{m s}^{-1}$. Find the velocity of P after the collision and the loss of kinetic energy due to the collision.

5 A particle P of mass $3m$ is moving in a straight line with speed u at the instant when it collides directly with a particle Q of mass m which is at rest. The coefficient of restitution between P and Q is e.

a Show that after the collision P is moving with speed $\frac{u(3-e)}{4}$ **(6 marks)**

b Show that the loss of kinetic energy due to the collision is $\frac{3mu^2(1-e^2)}{8}$ **(10 marks)**

c Find, in terms of m, u and e, the impulse exerted on Q by P in the collision. **(4 marks)**

6 Two spheres of masses 70 g and 100 g respectively are moving in opposite directions with speeds $4\,\text{m s}^{-1}$ and $8\,\text{m s}^{-1}$ respectively. The spheres collide directly. The coefficient of restitution between the spheres is $\frac{5}{12}$. Find:

a the velocities of the spheres after impact **(4 marks)**

b the amount of kinetic energy lost in the collision. **(3 marks)**

7 A sphere of mass 2 kg moving at $35\,\text{m s}^{-1}$ catches up and collides directly with a sphere of mass 10 kg moving in the same direction at $20\,\text{m s}^{-1}$. Five seconds after the impact the 10 kg sphere encounters a fixed barrier which reduces it to rest. Assuming the coefficient of restitution between the spheres is $\frac{3}{5}$, find the time that will elapse before the 2 kg sphere strikes the 10 kg sphere again. You may assume that the spheres are moving on a smooth surface and have constant speed between collisions. **(10 marks)**

8 Three balls A, B and C of masses $4m$, $3m$ and $3m$ respectively lie at rest on a smooth horizontal table with their centres in a straight line. The coefficient of restitution between any pair of balls is $\frac{3}{4}$. Show that if A is projected towards B with speed V there are three impacts and the final velocities are $\frac{5}{32}V$, $\frac{1}{4}V$ and $\frac{7}{8}V$ respectively. **(12 marks)**

9 A bullet of mass 60 g is fired horizontally at a fixed vertical metal barrier. The bullet hits the barrier when it is travelling at $600\,\text{m s}^{-1}$ and then rebounds.

a Find the kinetic energy lost at the impact if $e = 0.4$. **(8 marks)**

b Give one possible form of energy into which the lost kinetic energy has been transformed. **(1 mark)**

10 A particle A of mass $4m$ moving with speed u on a horizontal plane strikes directly a particle B of mass $3m$ which is at rest on the plane. The coefficient of restitution between A and B is e.

a Find, in terms of e and u, the speeds of A and B immediately after the collision. **(8 marks)**

b Given that the magnitude of the impulse exerted by A on B is $2mu$, show that $e = \frac{1}{6}$ **(6 marks)**

11 A ball of mass m moving with speed kV on a smooth table catches up and collides with another ball of mass λm moving with speed V travelling in the same direction on the table. The impact reduces the first ball to rest.

a Show that the coefficient of restitution is $\dfrac{\lambda + k}{\lambda(k-1)}$ **(6 marks)**

b Show further that $\lambda > \dfrac{k}{k-2}$ and $k > 2$. **(6 marks)**

12 A ball is dropped from zero velocity and after falling for 1 s under gravity meets another identical ball which is moving upwards at $7\,\text{m s}^{-1}$.

a Taking the value of g as $9.8\,\text{m s}^{-2}$, calculate the velocity of each ball immediately after the impact, given that the coefficient of restitution is $\frac{1}{4}$ **(8 marks)**

b Find the percentage loss in kinetic energy due to the impact, giving your answer to 2 significant figures. **(4 marks)**

13 A particle falls from a height 8 m onto a fixed horizontal plane. The coefficient of restitution between the particle and the plane is $\frac{1}{4}$

a Find the height to which the particle rises after impact. **(6 marks)**

b Find the time the particle takes from leaving the plane after impact to reach the plane again. **(2 marks)**

c Find the speed of the particle after the second rebound. **(2 marks)**

You may leave your answers in terms of g.

14 A particle falls from a height h onto a fixed horizontal plane. If e is the coefficient of restitution between the particle and the plane, show that the total time taken before the particle finishes bouncing is $\frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$ **(10 marks)**

15 A sphere P of mass m lies on a smooth table between a sphere Q of mass $8m$ and a fixed vertical plane. Sphere Q is initially at rest. Sphere P is projected towards sphere Q so they impact directly. The coefficient of restitution between the two spheres is $\frac{7}{8}$. Given that sphere P is reduced to rest by a second impact with sphere Q, find the coefficient of restitution between sphere P and the fixed vertical plane. **(12 marks)**

16 A gun of mass M kg is free to move horizontally. The gun fires a shell of mass m kg in a horizontal direction. The energy released by the explosion, which occurred in firing the shell, is E J. Find the velocity of the shell in terms of m, M and E if all of this energy is given to the shell and the gun. **(8 marks)**

17 A snooker ball of mass m kg is dropped from a point H m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of h m. Given that the coefficient of restitution between the ball and the floor is e,

a show that $e = \sqrt{\frac{h}{H}}$ **(6 marks)**

b find the height the ball reaches after it bounces a second time **(3 marks)**

c describe the subsequent motion of the ball according to this model. **(2 marks)**

18 A small sphere B of mass 2 kg is held at rest 2 m up a smooth slope that is angled at 30° to the horizontal. When sphere B is released from rest it rolls down the slope onto a smooth, horizontal plane where it collides directly with a stationary small sphere C of mass 1 kg. Given that the coefficient of restitution between the balls is 0.75, calculate:

a the speed and direction of motion of B and C after the collision **(12 marks)**

b the kinetic energy lost in the collision. **(5 marks)**

c Without doing any further calculations, state how the amount of kinetic energy lost in the collision would change if $e < 0.75$. **(1 mark)**

Problem-solving

Resolve to find the acceleration of B.

19 Two spheres A and B sit on a smooth horizontal plane at a point P between two parallel, vertical walls W_1 and W_2 such that the ratio of distance $W_1P : PW_2$ is 2 : 1. Sphere A is projected towards W_1 with speed 2 m s^{-1} and B is projected towards W_2 with speed 3 m s^{-1}. The spheres rebound off their respective walls before colliding at a point Q. Given that the coefficient of restitution between both walls and spheres is 0.6, calculate the ratio of the distances $W_1Q : W_2Q$. **(8 marks)**

Challenge

Three small spheres A, B and C of masses m_1, m_2 and m_3 respectively lie in order in a straight line on a large smooth table. Two slack strings connect A to B and B to C. Sphere C is projected with speed u m s^{-1} away from the other two spheres. Show that the total kinetic energy of A, B and C when both strings are taut is $\frac{m_3^2u^2}{2(m_1+m_2+m_3)}$