Paper Class – Probability

1.

The probability that Janice sees a kingfisher on any particular day is 0.3. She notes the number, X, of days in a week on which she sees a kingfisher.

(i) State one necessary condition for X to have a binomial distribution. **[1]**

Assume now that X has a binomial distribution.

(ii) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days. **[1]**

Each week Janice notes the number of days on which she sees a kingfisher.

(iii) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks. **[3]**

2.

A biased four-sided spinner has edges numbered 1, 2, 3, 4. When the spinner is spun, the probability that it will land on the edge numbered X is given by

$$P(X=x)=\begin{cases}\frac{1}{2}-\frac{1}{10}x & x=1,2,3,4,\\ 0 & \text{otherwise.}\end{cases}$$

(a) Draw a table showing the probability distribution of X. **[1]**

The spinner is spun three times and the value of X is noted each time.

(b) Find the probability that the third value of X is greater than the sum of the first two values of X. **[3]**

3.

The probability distribution of the discrete random variable X is given in Fig. 4.

r	0	1	2	3	4
$P(X=r)$	0.2	0.15	0.3	k	0.25

Fig. 4

(i) Find the value of k. **[2]**

X_1 and X_2 are two independent values of X.

(ii) Find $P(X_1+X_2=6)$. **[3]**

4.

A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty.

(a) Find the percentage of components bought from supplier C that are faulty.

(3)

A component is selected at random.

(b) Explain why the event "the component was bought from supplier B" is not statistically independent from the event "the component is faulty".

(1)

5.

A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4
Give your answer to 3 significant figures.

(3)

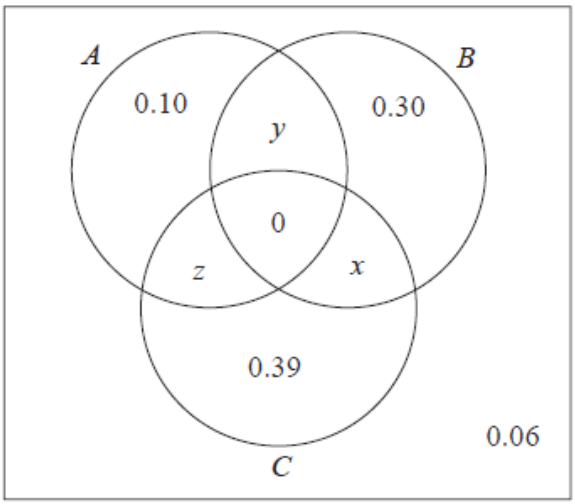
The random variable $Y = \dfrac{12}{X}$

(c) Find $P(Y - X \leqslant 4)$

(3)

6.

The Venn diagram shows three events, A, B and C, and their associated probabilities.



Events B and C are mutually exclusive.
Events A and C are independent.

Showing your working, find the value of x, the value of y and the value of z.

(5)

7.

A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leqslant X < 8)$

(5)

8.

In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(6)

9.

Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

10.

Two independent events, A and B, are such that

$$P(A) = 0.2$$

$$P(A \cup B) = 0.8$$

(a) (i) Find $P(B)$

[4 marks]

(a) (ii) Find $P(A \cap B)$

[1 mark]

(b) State, with a reason, whether or not the events A and B are mutually exclusive.

[1 mark]