Paper Class – Motion with Variable Acceleration

1.

A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

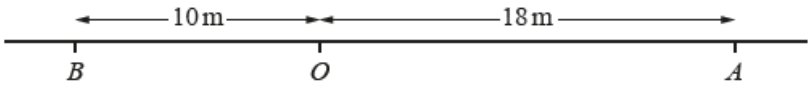
(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

2.



A particle P is moving along a straight line with constant acceleration. Initially the particle is at O. After 9 s, P is at a point A, where $OA = 18$ m (see diagram) and the velocity of P at A is $8\,\text{m s}^{-1}$ in the direction $\overrightarrow{OA}$.

(i) **(a)** Show that the initial speed of P is $4\,\text{m s}^{-1}$. **[2]**

(b) Find the acceleration of P. **[2]**

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

(ii) Show that P is never at point B. **[4]**

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O, and the displacement of Q from O at time t seconds is given by

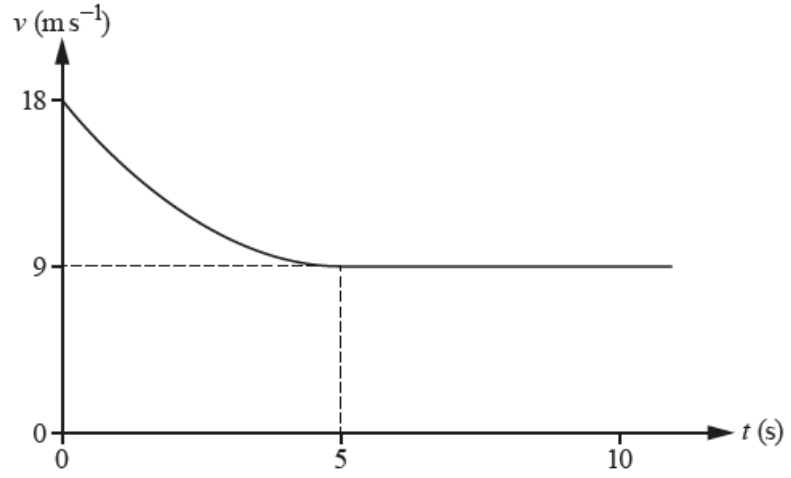
$$x = at^3 + bt^2 + ct,$$

where a, b and c are constants.

It is given that
- the velocity and acceleration of Q at the point O are the same as those of P at O,
- Q reaches the point A when $t = 6$.

(iii) Find the velocity of Q at A. **[5]**

3.



The diagram shows the velocity-time graph modelling the velocity of a car as it approaches, and drives through, a residential area.

The velocity of the car, $v\,\text{m s}^{-1}$, at time t seconds for the time interval $0 \leqslant t \leqslant 5$ is modelled by the equation $v = pt^2 + qt + r$, where p, q and r are constants.

It is given that the acceleration of the car is zero at $t = 5$ and the speed of the car then remains constant.

(a) Determine the values of p, q and r. **[5]**

(b) Calculate the distance travelled by the car from $t = 2$ to $t = 10$. **[3]**

4.

A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\text{m s}^{-1}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

5.

A car, starting from rest, is driven along a horizontal track.

The velocity of the car, $v\,\text{m s}^{-1}$, at time t seconds, is modelled by the equation

$$v = 0.48t^2 - 0.024t^3 \text{ for } 0 \leq t \leq 15$$

(a) Find the distance the car travels during the first 10 seconds of its journey.

[3 marks]

(b) Find the maximum speed of the car.

Give your answer to three significant figures.

[4 marks]

(c) Deduce the range of values of t for which the car is modelled as decelerating.

[2 marks]

6.

A particle P is moving in a straight line. At time t seconds P has velocity $v\,\text{m s}^{-1}$ where $v = (2t+1)(3-t)$.

(a) Find the deceleration of P when $t = 4$. **[2]**

(b) State the positive value of t for which P is instantaneously at rest. **[1]**

(c) Find the total distance that P travels between times $t = 0$ and $t = 4$. **[3]**

7.

In this question you should take g to be $10\,\text{m s}^{-2}$.

Piran finds a disused mineshaft on his land and wants to know its depth, d metres.

Local records state that the mineshaft is between 150 and 200 metres deep.

He drops a small stone down the mineshaft and records the time, T seconds, until he hears it hit the bottom. It takes 8.0 seconds.

Piran tries three models, A, B and C.

In model A, Piran uses the formula $d = 5T^2$ to estimate the depth.

(i) Find the depth that model A gives and comment on whether it is consistent with the local records.

Explain how the formula in model A is obtained. **[4]**

In model B, Piran uses the speed-time graph in Fig. 6.

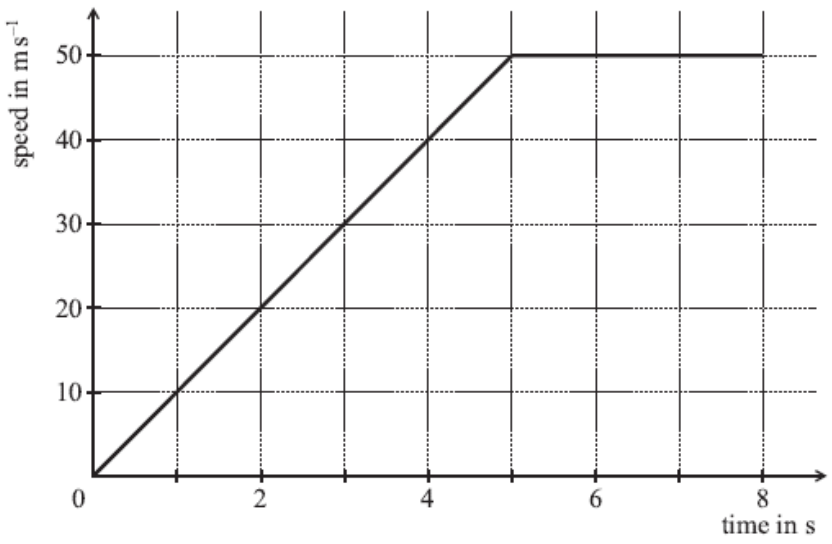


Fig. 6

(ii) Calculate the depth of the mineshaft according to model B.

Comment on whether this depth is consistent with the local records. **[4]**

(iii) Describe briefly one respect in which model B is the same as model A and one respect in which it is different. **[2]**

Piran then tries model C in which the speed, $v\,\mathrm{m\,s^{-1}}$, is given by

$$v = 10t - t^2 \text{ for } 0 \leqslant t \leqslant 5,$$

$$v = 25 \text{ for } 5 < t \leqslant 8.$$

(iv) Calculate the depth of the mineshaft according to model C.

Comment on whether this depth is consistent with the local records. **[6]**

(v) Describe briefly one respect in which model C is similar to model B and one respect in which it is different. **[2]**