Paper Class – Motion with Variable Acceleration – Mark Scheme

1.

Question	Scheme	Marks
(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1
	$v = 2t^3 - 3t^2 + t$	A1
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	**DM**1
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1
	All three	A1
		(5)
(b)	Find x when $t = 0$, $\frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1
		(3)
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso
		(2)

2.

Question			Answer	Marks
	(i)	(a)	$18=\left(\frac{8+u}{2}\right)(9)$	M1
			$u=-4$ therefore the speed of P is 4 (m s^{-1})	A1 [2]
	(i)	(b)	eg $8=-4+9a$	M1
			$a=\frac{4}{3}$ (m s^{-2})	A1 [2]

Question			Answer	Marks
	(ii)		$0=-4+\frac{4}{3}t$	M1
			$t=3$	A1
			$-s_{\max}=-4t+\frac{1}{2}\left(\frac{4}{3}\right)t^2$	M1
			$s_{\max}=6<10$ so P is never at B	A1 [4]
		OR		
				M1
			$-10=-4t+\frac{1}{2}\left(\frac{4}{3}\right)t^2$	A1
				M1
			e.g. det $=-24$ therefore not possible	A1
		OR		
			$0=(\pm4)^2+2\left(\frac{4}{3}\right)s$ or $v^2=(\pm4)^2+2\left(\frac{4}{3}\right)(-10)$	M2
			$s=-6$ or $v^2=-\frac{32}{3}$	A1
			Suitable conclusion	A1

Question	Answer	Marks
(iii)	$x = at^3 + bt^2 + ct$	
	$\dot{x} = 3at^2 + 2bt + c$	M1
	$\ddot{x} = 6at + 2b$	M1
	$c = -4$ and $b = \frac{2}{3}$	A1ft
	$18 = a(6)^3 + \frac{2}{3}(6)^2 - 4(6) \Rightarrow a = \frac{1}{12}$	A1ft
	Velocity of $Q = \left(\frac{1}{4}(6)^2 + \frac{4}{3}(6) - 4\right) = 13\,(\text{m s}^{-1})$	A1 [5]

3.

(a)	$v = pt^2 + qt + r$	
	$t = 0, v = 18 \Rightarrow r = 18$	B1
	$t = 5, v = 9 \Rightarrow 25p + 5q + 18 = 9$	M1
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 2pt + q$	B1
	$t = 5, \frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow 10p + q = 0$	M1
	$p = \frac{9}{25}, q = -\frac{18}{5}$	A1
		[5]
(b)	$\int_2^5 \left(\frac{9}{25}t^2 - \frac{18}{5}t + 18\right) \mathrm{d}t$	M1
	$+9 \times 5$	B1
	$= 75.24\,\text{m}$	A1
		[3]

4.

Question	Scheme	Marks	AOs
(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a
	$t = 6$ (or -2)	A1	1.1b
	Differentiate v wrt t	M1	1.1a
	$(a = \frac{dv}{dt} =) \ 4 - 2t$	A1	1.1b
	When $t = 6$, $a =$ - 8; Magnitude is 8 (m s^{-2})	A1	1.1b
		(5)	
(b)	Integrate v wrt t	M1	3.1a
	$(s =) 12t + 2t^2 - \frac{1}{3}t^3 (+C)$	A1	1.1b
	$t = 3$ => distance $= 45$ (m)	A1	1.1b
		(3)	

5.

Q	Marking Instructions	AO	Marks	Typical Solution
(a)	Integrates given velocity equation, with at least one term correct PI	3.4	M1	$s = \int_0^{10} 0.48t^2 - 0.024t^3 \, dt$
	Obtains fully correct integral PI	1.1b	A1	$s = [0.16t^3 - 0.006t^4]_0^{10}$
	Substitutes correct limits to obtain correct answer. CAO. Condone omission of units.	1.1b	A1	$s = 100$ m
(b)	Differentiates v (at least one term correct) PI	3.4	M1	$\frac{dv}{dt} = 0.96t - 0.072t^2$
	Equates their $\frac{dv}{dt}$ equation to zero PI	1.1a	M1	$0.96t - 0.072t^2 = 0$
	Finds correct non-zero value for t	1.1b	A1	$t = 0$ *or* $t = \frac{40}{3}$
	Finds correct maximum speed 28.4 Condone exact answer $28\frac{4}{9}$. Condone omission of units.	1.1b	A1	When $t = \frac{40}{3}$ then $v = 28.4$ ms^{-1}
(c)	Deduces lower critical value of t with correct associated inequality Follow through their value of t from part (b) provided $0 < t < 15$	2.2a	R1F	$\frac{40}{3} < t \leq 15$
	States $t \leq 15$	2.5	R1	

6.

Question			Answer	Marks
	(a)		Value of $\frac{dv}{dt}$ at $t = 4$ is -11 Deceleration of P is $11\ \text{m s}^{-2}$	B1 B1
			Alternative solution $\frac{dv}{dt} = 5 - 4t$ evaluated at $t = 4$ Deceleration of P is $11\ \text{m s}^{-2}$	M1 A1 [2]
	(b)		$t = 3\text{s}$	B1 [1]
	(c)		$\int_0^3 (2t+1)(3-t)\,dt = \frac{27}{2}$ $\int_3^4 (2t+1)(3-t)\,dt = -\frac{25}{6}$ Total distance is $\frac{27}{2} + \frac{25}{6} = \frac{53}{3}$ m or 17.7 m	B1 B1 B1 [3]

7.

Part	Answer	Mark
(i)	$d = 5\times 8^2 = 320$, so 320 m	B1
	This value is too great. It is not between 150 and 200 m.	B1
	$s = ut + \frac{1}{2}at^2$ with $s = d$, $(u = 0)$, $a = 10$ and $t = T$	M1
	Giving $d = \frac{1}{2}\times 10\times T^2 = 5T^2$	A1
		[4]
(ii)	Depth = Area under the graph	M1
	$= \frac{1}{2}\times 5\times 50 + 3\times 50$	A1
	= 275 m	A1
	Outside the 150 to 200 m interval so inconsistent	B1
		[4]
(iii)	The same: initial constant acceleration (of 10 m s^{-2})	B1
	Different: two part motion with constant speed at end	B1
		[2]

(iv)	For $0 \le t \le 5$, the distance travelled is $\int_0^5 (10t - t^2)\mathrm{d}t$	**M1**
	$= \left[5t^2 - \frac{t^3}{3}\right]_0^5$	**A1**
	$5 \times 5^2 - \frac{5^3}{3}$ $(= 83\frac{1}{3})$	**A1**
	For $5 < t \le 8$, the distance travelled is 25×3 $(= 75)$	**B1**
	$d = 83\frac{1}{3} + 75 = 158\frac{1}{3}$	**A1**
	This is within the given interval.	**B1**
		[6]
(v)	Similar: constant speed for $5 < t \le 8$	**B1**
	Different: acceleration is not constant for $0 \le t \le 5$.	**B1**
		[2]