Sequences 2 - Answers

1.

(a)	John; arithmetic series, $a = 60$, $d = 15$.		
	$60 + 75 + 90 = 225$* or $S_3 = \frac{3}{2}(120 + (3-1)(15)) = 225$*	Finds and adds the first 3 terms or uses sum of 3 terms of an AP and obtains the printed answer, with no errors.	B1 *
	Beware: **The 12th term of the sequence is 225 also so look out for** $60 + (12-1)\times 15 = 225$. **This is B0.**		
			[1]
(b)	$t_9 = 60 + (n-1)15 = (£)180$	M1: Uses $60 + (n-1)15$ with $n = 8$ or 9 A1: (£)180	M1 A1
	Listing: M1: Uses $a = 60$ and $d = 15$ to select the 8th or 9th term (allow arithmetic slips) A1: (£)180 (Special case (£)165 **only** scores M1A0)		
			[2]
(c)	$S_n = \frac{n}{2}(120 + (n-1)(15))$ or $S_n = \frac{n}{2}(60 + 60 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$ and $d = 15$ (must be a correct formula but ignore the value they use for n or could be in terms of n)	M1
	$S_n = \frac{12}{2}(120 + (12-1)(15))$	Correct numerical expression	A1
	$= (£)1710$	cao	A1
	Listing: M1: Uses $a = 60$ and $d = 15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: (£)1710		
			[3]
(d)	$3375 = \frac{n}{2}(120 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$, $d = 15$ and puts = 3375	M1
	$6750 = 15n(8 + (n-1)) \Rightarrow 15n^2 + 105n = 6750$	**Correct three term** quadratic. E.g. $6750 = 105n + 15n^2$, $3375 = \frac{15}{2}n^2 + \frac{105}{2}n$ This may be implied by equations such as $6750 = 15n(n+7)$ or $3375 = \frac{15}{2}(n^2 + 7n)$	A1
	$n^2 + 7n = 25\times 18$ *	Achieves the printed answer with no errors but must see the 450 or 450 in factorised form or e.g. 6750, 3375 in factorised form i.e. an intermediate step.	A1*
			[3]
(e)	$n = 18 \Rightarrow$ Aged 27	M1: Attempts to solve the given quadratic or states $n = 18$ A1: Age = 27 or just 27	M1 A1
	Age = 27 only scores both marks (i.e. $n = 18$ need not be seen)		
	Note that (e) is not hence so allow valid attempts to solve the given equation for M1		
			[2]

2.

	(a) Use $n^{th}\text{term} = a + (n-1)d$ with $d = 10$; $a = 150$ and $n = 8$, or $a = 160$ and $n = 7$, or $a = 170$ and $n = 6$: $= 150+7\times10$ or $160+6\times10$ or $170+5\times10$ $= 220$* (Or gives clear list – see note)	M1 A1*	(2)
Or	If answer 220 is assumed and $150 + (n-1)\,10 = 220$ or variation is solved for n= Then $n = 8$, so 2007 is the year (must conclude the year)	M1 A1*	(2)
	(b) Use $S_n = \frac{n}{2}\{2a+(n-1)10\}$ Or $S_n = \frac{n}{2}\{a+l\}$ **and** $l = a + (n-1)10$ $= 7(300+13\times10)$ or $7(150+280)$ $= 7\times430$ $= 3010$	M1 A1 A1	(3)
	(c) Cost in year n $= 900+(n-1)\times-20$ Sales in year $n = 150+(n-1)\times10$ Cost $= 3\times$Sales $\Rightarrow 900+(n-1)\times-20 = 3\times(150+(n-1)\times10)$ $900-20n+20 = 450+30n-30$ $500 = 50n$ $n = 10$ Year is 2009 As n is not defined they may work correctly from another base year to get the answer 2009 and their n may not equal 10. If doubtful – send to review.	M1 M1 M1 A1	(4)

3.

(a)	$600 = 200 + (N-1)20 \Rightarrow N = \ldots$	Use of 600 with a **correct** formula in an attempt to find N. A correct formula could be implied by a correct answer.	M1
	$N = 21$	cso	A1
	Accept correct answer only.		
	$600 = 200 + 20N \Rightarrow N = 20$ is M0A0 (wrong formula) $\frac{600-200}{20} = 20 \therefore N = 21$ is M1A1 (correct formula implied)		
	Listing: All terms must be listed up to 600 and 21 correctly identified. A solution that scores 2 if fully correct and 0 otherwise.		
			(2)
(b)	**Look for an AP first:**		
	$S = \frac{21}{2}(2\times200+20\times20)$ or $\frac{21}{2}(200+600)$ or $S = \frac{20}{2}(2\times200+19\times20)$ or $\frac{20}{2}(200+580)$ (= 8400 or 7800)	M1: Use of correct sum formula with their **integer** $n = N$ or $N-1$ from part (a) where $3 < N < 52$ and $a = 200$ and $d = 20$. A1: Any correct un-simplified numerical expression with $n = 20$ or $n = 21$ (No follow through here)	M1A1

	Then for the constant terms:		
	$600\times(52-"N")(=18600)$	M1: $600\times k$ where k is an integer and $3<k<52$ A1: A correct un-simplified follow through expression with their k consistent with n so that $n+k=52$	M1A1ft
	So total is 27000	Cao	A1
	Note that for the constant terms, they may correctly use an AP sum with $d=0$.		
	There are no marks in (b) for just finding S_{52}		
			(5)
			[7]
	If they obtain $N=20$ in (a) (0/2) and then in (b) proceed with, $S=\frac{20}{2}(2\times200+19\times20)+32\times600=7800+19\ 200=27\ 000$ allow them to 'recover' and score full marks in (b) Similarly If they obtain $N=22$ in (a) (0/2) and then in (b) proceed with, $S=\frac{21}{2}(2\times200+20\times20)+31\times600=8400+18\ 600=27\ 000$ allow them to 'recover' and score full marks in (b)		

4.

(i).(a)	$U_3=4$	cao	B1
			(1)
(b)	$\sum_{n=1}^{n=20}U_n=4+4+4.....+4$ or 20×4	For realising that all 20 terms are 4 and that the sum is required. Possible ways are 4+4+4…..+4 or 20×4 or $\frac{1}{2}\times20(2\times4+19\times0)$ or $\frac{1}{2}\times20(4+4)$ (Use of a correct sum formula with $n=20$, $a=4$ and $d=0$ or $n=20$, $a=4$ and $l=4$)	M1
	= 80	cao	A1
	Correct answer with no working scores M1A1		
			(2)
(ii)(a)	$V_3=3k,\quad V_4=4k$	May score in (b) if clearly identified as V_3 and V_4	B1, B1
			(2)
(b)	$\sum_{n=1}^{n=5}V_n=k+2k+3k+4k+5k=165$ or $\frac{1}{2}\times5(2\times k+4\times k)=165$ or $\frac{1}{2}\times5(k+5k)=165$	Attempts V_5, adds their V_1,V_2,V_3,V_4,V_5 AND sets equal to 165 or Use of a correct sum formula with $a=k$, $d=k$ and $n=5$ or $a=k$, $l=5k$ and $n=5$ AND sets equal to 165	M1

$15k = 165 \Rightarrow k = ..$	Attempts to solve their linear equation in k **having set the sum of their first 5 terms equal to 165.** Solving $V_5 =$ 165 scores no marks.	M1
$k = 11$	cao and cso	A1
		(3)

5.

	$a_1 = 4,\ a_{n+1} = 5 - ka_n,\ n \ldots 1$		
(a)	$a_2 = 5 - ka_1 = 5 - 4k$ $a_3 = 5 - ka_2 = 5 - k(5 - 4k)$	M1: Uses the recurrence relation correctly at least once. This may be implied by $a_2 = 5 - 4k$ or by the use of $a_3 = 5 - k(\text{their } a_2)$ A1: Two correct expressions – need not be simplified but must be seen in (a). Allow $a_2 = 5 - k4$ and $a_3 = 5 - 5k + k^2 4$ Isw if necessary for a_3.	M1A1
			[2]
(b)	$\sum_{r=1}^{3}(1) = 1+1+1$	Finds 1+1+1 or 3 somewhere in their solution (may be implied by e.g. $5 + 6 - 4k + 6 - 5k + 4k^2$). Note that $5 + 6 - 4k + 6 - 5k + 4k^2$ would score B1 and the M1 below.	B1
	$\sum_{r=1}^{3} a_r = 4 + "5 - 4k" + "5 - 5k + 4k^2"$	Adds 4 to their a_2 and their a_3 where a_2 and a_3 are functions of k. The statement as shown is sufficient.	M1
	$\sum_{r=1}^{3}(1 + a_r) = 17 - 9k + 4k^2$	Cao but condone '= 0' after the expression	A1
	Allow full marks in (b) for correct answer only		
			[3]
(c)	500	cao	B1
			[1]

6.

(a) $(S =)\ a + ar + \ldots + ar^{n-1}$	"$S =$" not required.	Addition required.	B1
$(rS =)\ ar + ar^2 + \ldots + ar^n$	"$rS =$" not required	(M: Multiply by r)	M1
$S(1-r) = a(1-r^n)$ $\quad S = \frac{a(1-r^n)}{1-r}$	(M: Subtract and factorise) (*)		M1 A1cso (4)
(b) $ar^{n-1} = 35000 \times 1.04^3 = 39400$	(M: Correct a and r, with n = 3, 4 or 5).		M1 A1 (2)
(c) $n = 20$	(Seen or implied)		B1

$S_{20} = \dfrac{35000(1-1.04^{20})}{(1-1.04)}$ (M1: Needs <u>any</u> r value, $a = 35000$, n =19, 20 or 21). (A1ft: ft from n =19 or n =21, but r must be 1.04). $= 1\ 042\ 000$	M1 A1ft A1 (4) **10**
(a) B1: At least the 3 terms shown above, and no extra terms. A1: Requires a completely correct solution. <u>Alternative for the 2 M marks</u>: M1: Multiply numerator and denominator by $1 - r$. M1: Multiply out numerator convincingly, and factorise. (b) M1 can also be scored by a "year by year" method. <u>Answer only:</u> 39 400 scores full marks, 39 370 scores M1 A0. (c) M1 can also be scored by a "year by year" method, <u>with terms added</u>. In this case the B1 will be scored if the correct number of years is considered. <u>Answer only:</u> Special case: 1 042 000 scores 2 B marks, scored as 1, 0, 0, 1 (Other answers score no marks). <u>Failure to round correctly in (b) and (c):</u> Penalise once only (first occurrence).	

7.

(a)

$\{S_n =\}\ a + ar + \ldots + ar^{n-1}$	B1
$\{rS_n =\}\ ar + ar^2 + \ldots + ar^n$ $(1-r)S_n = a(1-r^n)$	M1 **d**M1
$S_n = \dfrac{a(1-r^n)}{1-r}$ (✱)	A1cso (4)

(b)

$a = 200,\ r = 2,\ n = 10,\quad S_{10} = \dfrac{200(1-2^{10})}{1-2}$	M1, A1
$= 204{,}600$	A1 (3)

(c)

$a = \dfrac{5}{6},\ r = \dfrac{1}{3}$	B1
$S_\infty = \dfrac{a}{1-r},\qquad S_\infty = \dfrac{\frac{5}{6}}{1-\frac{1}{3}}$	M1
$= \dfrac{5}{4}$ o.e.	A1 (3)

(d)

$-1 < r < 1$ (or $\lvert r \rvert < 1$)	B1 (1) **(11)**

8.

(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))	B1	(1)
(b) $50\,000r^{n-1} > 200000$ (Using answer to (a), which must include r and n, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)	M1	
$r^{n-1} > 4 \quad \Rightarrow \quad (n-1)\log r > \log 4$ (Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)	M1	
$n > \dfrac{\log 4}{\log r} + 1$ (*)	A1cso	(3)
(c) $r = 1.09$: $n > \dfrac{\log 4}{\log 1.09} + 1$ or $n - 1 > \dfrac{\log 4}{\log 1.09}$ $(n > 17.086...)$ (Allow equality)	M1	
Year 18 or 2023 (If one of these is correct, ignore the other)	A1	(2)
(d) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{50000(1-1.09^{10})}{1-1.09}$	M1 A1	
£760 000 (Must be this answer… nearest £10000)	A1	(3)
		9