

Momentum and Impulse

Exercise A

- 1 A ball of mass 0.5 kg is at rest when it is struck by a bat and receives an impulse of 15 N s . Find its speed immediately after it is struck.
 - 2 A ball of mass 0.3 kg moving along a horizontal surface hits a fixed vertical wall at right angles with speed 3.5 m s^{-1} . The ball rebounds at right angles to the wall. Given that the magnitude of the impulse exerted on the ball by the wall is 1.8 N s , find the speed of the ball just after it rebounds.
 - 3 A toy car of mass 0.2 kg is pushed from rest along a smooth horizontal floor by a horizontal force of magnitude 0.4 N for 1.5 s . Find its speed at the end of the 1.5 s .
 - 4 A ball of mass 0.2 kg , moving along a horizontal surface, hits a fixed vertical wall at right angles. The ball rebounds at right angles to the wall with speed 3.5 m s^{-1} . Given that the magnitude of the impulse exerted on the ball by the wall is 2 N s , find the speed of the ball just before it hits the wall. (3 marks)
 - 5 A ball of mass 0.2 kg is dropped from a height of 2.5 m above horizontal ground. After hitting the ground it bounces to a height of 1.8 m above the ground. Find the magnitude of the impulse received by the ball from the ground. (4 marks)
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Exercise B

- 1 A particle P of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1} . It collides with a second particle Q of mass 1 kg which is at rest. After the collision P has speed 2 m s^{-1} and it continues to move in the same direction. Find the speed of Q after the collision.
- 2 A railway truck of mass 25 tonnes moving at 4 m s^{-1} collides with a stationary truck of mass 20 tonnes . As a result of the collision the trucks couple together. Find the common speed of the trucks after the collision.
- 3 Particles A and B have masses 0.5 kg and 0.2 kg respectively. They are moving with speeds 5 m s^{-1} and 2 m s^{-1} respectively in the same direction along the same straight line on a smooth horizontal surface when they collide. After the collision A continues to move in the same direction with speed 4 m s^{-1} . Find the speed of B after the collision.
- 4 A particle of mass 2 kg is moving on a smooth horizontal plane with speed 4 m s^{-1} . It collides with a second particle of mass 1 kg which is at rest. After the collision the particles join together.
 - a Find the common speed of the particles after the collision.
 - b Find the magnitude of the impulse in the collision.
- 5 Two particles A and B of masses 2 kg and 5 kg respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of A and B are 6 m s^{-1} and 4 m s^{-1} respectively. After the collision the direction of motion of A is reversed and its speed is 1.5 m s^{-1} . Find:
 - a the speed and direction of B after the collision (3 marks)
 - b the magnitude of the impulse given by A to B in the collision. (3 marks)

- 6 A particle P of mass 150 g is at rest on a smooth horizontal plane. A second particle Q of mass 100 g is projected along the plane with speed $u\text{ m s}^{-1}$ and collides directly with P . On impact the particles join together and move on with speed 4 m s^{-1} . Find the value of u . (4 marks)
- 7 A particle A of mass $4m$ is moving along a smooth horizontal surface with speed $2u$. It collides with another particle B of mass $3m$ which is moving with the same speed along the same straight line but in the opposite direction. Given that A is brought to rest by the collision, find:
- the velocity of B after the collision (3 marks)
 - the magnitude of the impulse given by A to B in the collision. (3 marks)
- 8 An explosive charge of mass 150 g is designed to split into two parts, one with mass 100 g and the other with mass 50 g . When the charge is moving at 4 m s^{-1} it splits and the larger part continues to move in the same direction whilst the smaller part moves in the opposite direction. Given that the speed of the larger part is twice the speed of the smaller part, find the speeds of each of the two parts. (3 marks)
- 9 Two particles P and Q of masses m and km respectively are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of P and Q are $3u$ and u respectively. After the collision the direction of motion of both particles is reversed and the speed of each particle is halved.
- Find the value of k . (4 marks)
 - Find, in terms of m and u , the magnitude of the impulse given by P to Q in the collision. (3 marks)
- 10 Two particles A and B of masses 4 kg and 2 kg respectively are connected by a light inextensible string. The particles are at rest on a smooth horizontal plane with the string slack. Particle A is projected directly away from B with speed $u\text{ m s}^{-1}$. When the string goes taut the impulse transmitted through the string has magnitude 6 N s . Find:
- the common speed of the particles just after the string goes taut (4 marks)
 - the value of u . (3 marks)
- 11 Two particles P and Q of masses 3 kg and 2 kg respectively are moving along the same straight line on a smooth horizontal surface. The particles collide. After the collision both the particles are moving in the same direction, the speed of P is 1 m s^{-1} and the speed of Q is 1.5 m s^{-1} . The magnitude of the impulse of P on Q is 9 N s . Find:
- the speed and direction of P before the collision (3 marks)
 - the speed and direction of Q before the collision. (3 marks)
- 12 Two particles A and B are moving in the same direction along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speed of B is 1.5 m s^{-1} . After the collision the direction of motion of both particles is unchanged, the speed of A is 2.5 m s^{-1} and the speed of B is 3 m s^{-1} . Given that the mass of A is three times the mass of B ,
- find the speed of A before the collision. (4 marks)
- Given that the magnitude of the impulse on A in the collision is 3 N s ,
- find the mass of A . (3 marks)

Challenge

Particle P has mass $3m\text{ kg}$ and particle Q has mass $m\text{ kg}$. The particles are moving in opposite directions along the same straight line on a smooth horizontal plane when they collide directly. Immediately before the collision, the speed of P is $u_1\text{ m s}^{-1}$ and the speed of Q is $u_2\text{ m s}^{-1}$. In the collision, the direction of motion of P is unchanged and the direction of Q is reversed. Immediately after the collision, the speed of P is $\frac{1}{4}u_1$ and the speed of Q is $\frac{1}{2}u_2$. Show that $u_1 = \frac{2}{3}u_2$.

Exercise C

In this exercise $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

- 1 A particle of mass 0.25 kg is moving with velocity $(12\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ when it receives an impulse $(8\mathbf{i} - 7\mathbf{j}) \text{ N s}$. Find the new velocity of the particle.
- 2 A particle of mass 0.5 kg is moving with velocity $(2\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ when it receives an impulse $(3\mathbf{i} + 5\mathbf{j}) \text{ N s}$. Find the new velocity of the particle.
- 3 A particle of mass 2 kg moves with velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ immediately after it has received an impulse $(4\mathbf{i} + 8\mathbf{j}) \text{ N s}$. Find the original velocity of the particle.
- 4 A particle of mass 1.5 kg moves with velocity $(5\mathbf{i} - 8\mathbf{j}) \text{ m s}^{-1}$ immediately after it has received an impulse $(3\mathbf{i} - 6\mathbf{j}) \text{ N s}$. Find the original velocity of the particle.
- 5 A body of mass 3 kg is initially moving with a constant velocity of $(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$ when it is acted on by a force of $(6\mathbf{i} - 8\mathbf{j}) \text{ N}$ for 3 seconds. Find the impulse exerted on the body and find its velocity when the force ceases to act.
- 6 A body of mass 0.5 kg is initially moving with a constant velocity of $(5\mathbf{i} + 12\mathbf{j}) \text{ m s}^{-1}$ when it is acted on by a force of $(2\mathbf{i} - \mathbf{j}) \text{ N}$ for 5 seconds. Find the impulse exerted on the body and find its velocity when the force ceases to act.
- 7 A particle of mass 2 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$ when it hits a wall. It rebounds with velocity $(-\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$. Find the impulse exerted by the wall on the particle.
- 8 A particle of mass 0.5 kg is moving with velocity $(11\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ when it hits a wall. It rebounds with velocity $(-\mathbf{i} + 7\mathbf{j}) \text{ m s}^{-1}$. Find the impulse exerted by the wall on the particle.
- 9 A particle P of mass 3 kg receives an impulse $\mathbf{Q} \text{ N s}$. Immediately before the impulse the velocity of P is $5\mathbf{i} \text{ m s}^{-1}$ and immediately afterwards it is $(13\mathbf{i} - 6\mathbf{j}) \text{ m s}^{-1}$. Find the magnitude of $\mathbf{Q}$ and the angle between $\mathbf{Q}$ and $\mathbf{i}$.
- 10 A particle P of mass 0.5 kg receives an impulse $\mathbf{Q} \text{ N s}$. Immediately before the impulse the velocity of P is $(-\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ and immediately afterwards it is $(3\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$. Find the magnitude of $\mathbf{Q}$ and the angle between $\mathbf{Q}$ and $\mathbf{i}$.
- 11 A cricket ball of mass 0.5 kg is hit by a bat. Immediately before being hit the velocity of the ball is $(20\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$ and immediately afterwards it is $(-16\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-1}$. Find the magnitude of the impulse exerted on the ball by the bat. **(3 marks)**
- 12 A ball of mass 0.2 kg is hit by a bat. Immediately before being hit by the bat the velocity of the ball is $-15\mathbf{i} \text{ m s}^{-1}$ and the bat exerts an impulse of $(2\mathbf{i} + 6\mathbf{j}) \text{ N s}$ on the ball. Find the velocity of the ball after the impact. **(3 marks)**
- 13 A particle of mass 0.25 kg has velocity $\mathbf{v} \text{ m s}^{-1}$ at time $t \text{ s}$ where $\mathbf{v} = (t^2 - 3)\mathbf{i} + 4t\mathbf{j}$, $t \leq 3$. When $t = 3$, the particle receives an impulse of $(2\mathbf{i} + 2\mathbf{j}) \text{ N s}$. Find the velocity of the particle immediately after the impulse. **(3 marks)**
- 14 A ball of mass 2 kg is initially moving with a velocity of $(\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. It receives an impulse of $2\mathbf{j} \text{ N s}$. Find the velocity immediately after the impulse and the angle through which the ball is deflected as a result. Give your answer to the nearest degree. **(5 marks)**
- 15 A particle of mass 0.5 kg moving with velocity $3\mathbf{i} \text{ m s}^{-1}$ collides with a particle of mass 0.25 kg moving with velocity $12\mathbf{i} \text{ m s}^{-1}$. The two particles coalesce and move as one particle of mass 0.75 kg. Find the velocity of the combined particle. **(3 marks)**
- 16 A particle of mass 5 kg moving with velocity $(\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ collides with a particle of mass 2 kg moving with velocity $(-\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$. The two particles coalesce and move as one particle of mass 7 kg. Find the magnitude of the velocity $\mathbf{v} \text{ m s}^{-1}$ of the combined particle. **(3 marks)**

Challenge

A particle of mass m kg moves with a constant velocity of $(a\mathbf{i} + b\mathbf{j}) \text{ m s}^{-1}$. After being given an impulse, the particle then moves with a constant velocity of $(c\mathbf{i} + d\mathbf{j}) \text{ m s}^{-1}$. Given that the direction of the impulse makes an angle of 45° above the direction of $\mathbf{i}$, show that $b + c = a + d$.

Exercise D

- 1 A particle P of mass $3m$ is moving along a straight line with constant speed $2u$. It collides with another particle Q of mass $4m$ which is moving with speed u along the same line but in the opposite direction. As a result of the collision P is brought to rest.
 - a Find the speed of Q after the collision and state its direction of motion.
 - b Find the magnitude of the impulse exerted by Q on P in the collision.
- 2 A pile driver of mass 1000 kg drives a pile of mass 200 kg vertically into the ground. The driver falls freely a vertical distance of 10 m before hitting the pile. Immediately after the driver impacts with the pile it can be assumed that they both move with the same velocity. By modelling the pile and the driver as particles, find:
 - a the speed of the driver immediately before it hits the pile (2 marks)
 - b the common speed of the pile and driver immediately after the impact. (3 marks)The ground provides a constant resistance to the motion of the pile driver of magnitude $120\,000 \text{ N}$.
 - c Find the distance that the pile is driven into the ground before coming to rest. (2 marks)
 - d Comment on this model in relation to the motion of the pile and driver immediately after impact. (1 mark)
- 3 A car of mass 800 kg is travelling along a straight horizontal road. A constant retarding force of $F \text{ N}$ reduces the speed of the car from 18 m s^{-1} to 12 m s^{-1} in 2.4 s . Calculate:
 - a the value of F (4 marks)
 - b the distance moved by the car in these 2.4 s . (3 marks)
- 4 Two particles A and B , of masses 0.2 kg and 0.3 kg respectively, are free to move in a smooth horizontal groove. Initially B is at rest and A is moving toward B with a speed of 4 m s^{-1} . After the impact the speed of B is 1.5 m s^{-1} . Find:
 - a the speed of A after the impact (3 marks)
 - b the magnitude of the impulse of B on A during the impact. (3 marks)
- 5 A railway truck P of mass 2000 kg is moving along a straight horizontal track with speed 10 m s^{-1} . The truck P collides with a truck Q of mass 3000 kg , which is at rest on the same track. Immediately after the collision Q moves with speed 5 m s^{-1} . Calculate:
 - a the speed of P immediately after the collision (3 marks)
 - b the magnitude of the impulse exerted by P on Q during the collision. (3 marks)
- 6 A particle P of mass 1.5 kg is moving along a straight horizontal line with speed 3 m s^{-1} . Another particle Q of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 m s^{-1} . The particles collide. Immediately after the collision the direction of motion of P is reversed and its speed is 2.5 m s^{-1} .
 - a Calculate the speed of Q immediately after the impact. (3 marks)
 - b State whether or not the direction of motion of Q is changed by the collision. (1 mark)
 - c Calculate the magnitude of the impulse exerted by Q on P , giving the units of your answer. (3 marks)

- 7 A particle A of mass m is moving with speed $2u$ in a straight line on a smooth horizontal table. It collides with another particle B of mass km which is moving in the same straight line on the table with speed u in the opposite direction to A . In the collision, the particles form a single particle which moves with speed $\frac{2}{3}u$ in the original direction of A 's motion. Find the value of k . (3 marks)
- 8 A metal pin of mass 2 kg is driven vertically into the ground by a blow from a sledgehammer of mass 10 kg . The hammer falls vertically on to the pin, its speed just before impact being 9 m s^{-1} . In a model of the situation it is assumed that, after impact, the pin and the hammer stay in contact and move together before coming to rest.
- a Find the speed of the pin immediately after impact. (3 marks)
- The pin moves 3 cm into the ground before coming to rest. Assuming in this model that the ground exerts a constant resistive force of magnitude R newtons as the pin is driven down,
- b find the value of R . (5 marks)
- c State one way in which this model might be refined to be more realistic. (1 mark)
- 9 A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck the velocity of the ball is $-25\mathbf{i}\text{ m s}^{-1}$. Immediately after being struck the velocity of the ball is $(23\mathbf{i} + 20\mathbf{j})\text{ m s}^{-1}$. Find the magnitude of the impulse exerted on the ball by the bat and the angle between the impulse and the direction of $\mathbf{i}$. (5 marks)
- 10 A ball of mass 0.2 kg is hit by a bat which gives it an impulse of $(2.4\mathbf{i} + 3.6\mathbf{j})\text{ N s}$. The velocity of the ball immediately after being hit is $(12\mathbf{i} + 5\mathbf{j})\text{ m s}^{-1}$. Find the velocity of the ball immediately before it is hit. (3 marks)
- 11 A body P of mass 4 kg is moving with velocity $(2\mathbf{i} + 16\mathbf{j})\text{ m s}^{-1}$ when it collides with a body Q of mass 3 kg moving with velocity $(-\mathbf{i} - 8\mathbf{j})\text{ m s}^{-1}$. Immediately after the collision the velocity of P is $(-4\mathbf{i} - 32\mathbf{j})\text{ m s}^{-1}$. Find the velocity of Q immediately after the collision. (3 marks)
- 12 A particle P of mass 0.3 kg is moving so that its position vector $\mathbf{r}$ metres at time t seconds is given by
- $$\mathbf{r} = (t^3 + t^2 + 4t)\mathbf{i} + (11t)\mathbf{j}, \quad t \leq 4.$$
- a Calculate the speed of P when $t = 4$. (4 marks)
- When $t = 4$, the particle is given an impulse $(2.4\mathbf{i} + 3.6\mathbf{j})\text{ N s}$.
- b Find the velocity of P immediately after the impulse. (3 marks)

Challenge

A particle P of mass $m\text{ kg}$ moves at a speed of $u\text{ m s}^{-1}$ when it collides head on with a particle Q of mass $km\text{ kg}$ also travelling at a speed of $u\text{ m s}^{-1}$, travelling along the same line.

After collision, the two particles move off together with a common speed of $v\text{ m s}^{-1}$.

Show that k can be written as $\frac{u-v}{u+v}$ or $\frac{u+v}{u-v}$ and explain how these expressions relate to:

- a the relative size of u and v
- b the subsequent motion of P and Q after collision.