

Graphs

Exercise A

1 Sketch the following curves and indicate clearly the points of intersection with the axes:

a $y = (x-3)(x-2)(x+1)$

b $y = (x-1)(x+2)(x+3)$

c $y = (x+1)(x+2)(x+3)$

d $y = (x+1)(1-x)(x+3)$

e $y = (x-2)(x-3)(4-x)$

f $y = x(x-2)(x+1)$

g $y = x(x+1)(x-1)$

h $y = x(x+1)(1-x)$

i $y = (x-2)(2x-1)(2x+1)$

j $y = x(2x-1)(x+3)$

2 Sketch the curves with the following equations:

a $y = (x+1)^2(x-1)$

b $y = (x+2)(x-1)^2$

c $y = (2-x)(x+1)^2$

d $y = (x-2)(x+1)^2$

e $y = x^2(x+2)$

f $y = (x-1)^2x$

g $y = (1-x)^2(3+x)$

h $y = (x-1)^2(3-x)$

i $y = x^2(2-x)$

j $y = x^2(x-2)$

3 Factorise the following equations and then sketch the curves:

a $y = x^3 + x^2 - 2x$

b $y = x^3 + 5x^2 + 4x$

c $y = x^3 + 2x^2 + x$

d $y = 3x + 2x^2 - x^3$

e $y = x^3 - x^2$

f $y = x - x^3$

g $y = 12x^3 - 3x$

h $y = x^3 - x^2 - 2x$

i $y = x^3 - 9x$

j $y = x^3 - 9x^2$

4 Sketch the following curves and indicate the coordinates of the points where the curves cross the axes:

a $y = (x-2)^3$

b $y = (2-x)^3$

c $y = (x-1)^3$

d $y = (x+2)^3$

e $y = -(x+2)^3$

f $y = (x+3)^3$

g $y = (x-3)^3$

h $y = (1-x)^3$

i $y = -(x-2)^3$

j $y = -(x-\frac{1}{2})^3$

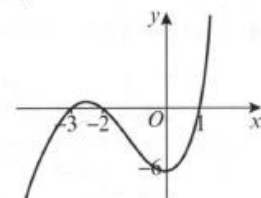
5 The graph of $y = x^3 + bx^2 + cx + d$ is shown opposite, where b , c and d are real constants.

a Find the values of b , c and d .

(3 marks)

b Write down the coordinates of the point where the curve crosses the y -axis.

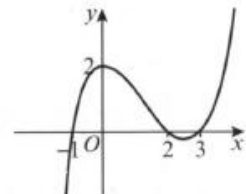
(1 mark)



6 The graph of $y = ax^3 + bx^2 + cx + d$ is shown opposite, where a , b , c and d are real constants.

Find the values of a , b , c and d .

(4 marks)



7 Given that $f(x) = (x-10)(x^2-2x) + 12x$

a Express $f(x)$ in the form $x(ax^2 + bx + c)$ where a , b and c are real constants.

(3 marks)

b Hence factorise $f(x)$ completely.

(2 marks)

c Sketch the graph of $y = f(x)$ showing clearly the points where the graph intersects the axes.

(3 marks)

Exercise B

1 Sketch the following curves and indicate clearly the points of intersection with the axes:

- a** $y = (x+1)(x+2)(x+3)(x+4)$ **b** $y = x(x-1)(x+3)(x-2)$
c $y = x(x+1)^2(x+2)$ **d** $y = (2x-1)(x+2)(x-1)(x-2)$
e $y = x^2(4x+1)(4x-1)$ **f** $y = -(x-4)^2(x-2)^2$
g $y = (x-3)^2(x+1)^2$ **h** $y = (x+2)^3(x-3)$
i $y = -(2x-1)^3(x+5)$ **j** $y = (x+4)^4$

Hint In part **f** the coefficient of x^4 will be negative.

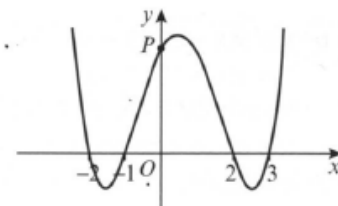
2 Sketch the following curves and indicate clearly the points of intersection with the axes:

- a** $y = (x+2)(x-1)(x^2-3x+2)$ **b** $y = (x+3)^2(x^2-5x+6)$
c $y = (x-4)^2(x^2-11x+30)$ **d** $y = (x^2-4x-32)(x^2+5x-36)$

Hint Factorise the quadratic factor first.

3 The graph of $y = x^4 + bx^3 + cx^2 + dx + e$ is shown opposite, where b, c, d and e are real constants.

- a** Find the coordinates of point P . (2 marks)
b Find the values of b, c, d and e . (3 marks)



4 Sketch the graph of $y = (x+5)(x-4)(x^2+5x+14)$. (3 marks)

Exercise C

1 Use a separate diagram to sketch each pair of graphs.

- a** $y = \frac{2}{x}$ and $y = \frac{4}{x}$ **b** $y = \frac{2}{x}$ and $y = -\frac{2}{x}$ **c** $y = -\frac{4}{x}$ and $y = -\frac{2}{x}$
d $y = \frac{3}{x}$ and $y = \frac{8}{x}$ **e** $y = -\frac{3}{x}$ and $y = -\frac{8}{x}$

2 Use a separate diagram to sketch each pair of graphs.

- a** $y = \frac{2}{x^2}$ and $y = \frac{5}{x^2}$ **b** $y = \frac{3}{x^2}$ and $y = -\frac{3}{x^2}$ **c** $y = -\frac{2}{x^2}$ and $y = -\frac{6}{x^2}$

Exercise D

1 In each case:

- i** sketch the two curves on the same axes
ii state the number of points of intersection
iii write down a suitable equation which would give the x -coordinates of these points.
 (You are not required to solve this equation.)

- a** $y = x^2, y = x(x^2-1)$ **b** $y = x(x+2), y = -\frac{5}{x}$ **c** $y = x^2, y = (x+1)(x-1)^2$
d $y = x^2(1-x), y = -\frac{2}{x}$ **e** $y = x(x-4), y = \frac{1}{x}$ **f** $y = x(x-4), y = -\frac{1}{x}$
g $y = x(x-4), y = (x-2)^3$ **h** $y = -x^3, y = -\frac{2}{x}$ **i** $y = -x^3, y = x^2$
j $y = -x^3, y = -x(x+2)$ **k** $y = 4, y = x(x-1)(x+2)^2$ **l** $y = x^3, y = x^2(x+1)^2$

- 2 a On the same axes sketch the curves given by $y = x^2(x - 3)$ and $y = \frac{c}{x}$
 b Explain how your sketch shows that there are only two real solutions to the equation $x^3(x - 3) = 2$.
- 3 a On the same axes sketch the curves given by $y = (x + 1)^3$ and $y = 3x(x - 1)$.
 b Explain how your sketch shows that there is only one real solution to the equation $x^3 + 6x + 1 = 0$.
- 4 a On the same axes sketch the curves given by $y = \frac{1}{x}$ and $y = -x(x - 1)^2$.
 b Explain how your sketch shows that there are no real solutions to the equation $1 + x^2(x - 1)^2 = 0$.
- 5 a On the same axes sketch the curves given by $y = x^2(x - a)$ and $y = \frac{b}{x}$ where a and b are both positive constants. (5 marks)
 b Using your sketch, state, giving a reason, the number of real solutions to the equation $x^4 - ax^3 - b = 0$. (1 mark)
- 6 a On the same set of axes sketch the graphs of $y = \frac{4}{x^2}$ and $y = 3x + 7$. (3 marks)
 b Write down the number of real solutions to the equation $\frac{4}{x^2} = 3x + 7$. (1 mark)
 c Show that you can rearrange the equation to give $(x + 1)(x + 2)(3x - 2) = 0$. (2 marks)
 d Hence determine the exact coordinates of the points of intersection. (3 marks)
- 7 a On the same axes sketch the curve $y = x^3 - 3x^2 - 4x$ and the line $y = 6x$.
 b Find the coordinates of the points of intersection.
- 8 a On the same axes sketch the curve $y = (x^2 - 1)(x - 2)$ and the line $y = 14x + 2$.
 b Find the coordinates of the points of intersection.
- 9 a On the same axes sketch the curves with equations $y = (x - 2)(x + 2)^2$ and $y = -x^2 - 8$.
 b Find the coordinates of the points of intersection.
- 10 a Sketch the graphs of $y = x^2 + 1$ and $2y = x - 1$. (3 marks)
 b Explain why there are no real solutions to the equation $2x^2 - x + 3 = 0$. (2 marks)
 c Work out the range of values of a such that the graphs of $y = x^2 + a$ and $2y = x - 1$ have two points of intersection. (5 marks)
- 11 a Sketch the graphs of $y = x^2(x - 1)(x + 1)$ and $y = \frac{1}{3}x^3 + 1$. (5 marks)
 b Find the number of real solutions to the equation $3x^2(x - 1)(x + 1) = x^3 + 3$. (1 mark)

Problem-solving

Even though you don't know the values of a and b , you know they are positive, so you know the shapes of the graphs. You can label the point a on the x -axis on your sketch of $y = x^2(x - a)$.