Functions - Answers

1.

(a) One-to-one
(b) One-to-one
(c) One-to-many
(d) Many-to-many
(e) Many-to-one
(f) Many-to-one

2.

(a)	$f(x) \geq 0$	allow $y \geq 0$	M1 A1	2	> 0 or $f \geq 0$ or ≥ 0
(b)(i)	$\sqrt{\frac{1}{x} - 2}$		B1	1	
(ii)	$\frac{1}{x} - 2 = 1$		M1		squaring their (b)(i) in an equation
	$\frac{1}{x} = 3$	OE	A1		
	$x = \frac{1}{3}$		A1	3	CSO
(c)	$y = \sqrt{x-2}$				
	$y^2 = x - 2$		M1		attempt to isolate; condone 1 slip
	$x^2 = y - 2$		M1		reverse $x \Leftrightarrow y$
	$y = x^2 + 2$		A1	3	

3.

(a)	$0 < f(x) < \frac{4}{5}$	M1A1 **(2)**
(b)	$y = \frac{4}{3x+5} \quad \Rightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \frac{4-5y}{3y}$	dM1
	$f^{-1}(x) = \frac{4-5x}{3x} \quad \left(0 < x < \frac{4}{5}\right)$	A1o.e. **(3)**
(c)	$fg(x) = \frac{4}{\frac{3}{x}+5}$	B1 **(1)**
(d)	$\frac{3x+5}{4} = \frac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4\times15\times15$ and so deduce no real roots	M1 A1 **(4)**

4.

(a)	$f(x) \geqslant 0$	M1 A1	2	For $\geqslant 0$, $f(x) > 0$ Correct; allow $y \geqslant 0$, $f \geqslant 0$
(b)(i)	$y = \sqrt{2x+5}$			
	$x = \sqrt{2y+5}$	M1		$x \Leftrightarrow y$
	$x^2 = 2y+5$	M1		Attempt to isolate, squaring first
	$f^{-1}(x) = \frac{x^2-5}{2}$	A1	3	condone $(y = \ )$
(ii)	$x \geqslant 0$	B1F	1	ft their (a), but **must** be x
(c)(i)	$h(x) = fg(x)$			
	$= \sqrt{2\left(\frac{1}{4x+1}\right)+5}$	B1	1	
(ii)	$\sqrt{2\left(\frac{1}{4x+1}\right)+5} = 3$			
	$2\left(\frac{1}{4x+1}\right)+5 = 9$	M1		one correct step from (c)(i), squaring
	$\frac{1}{4x+1} = 2$ or $4x+1 = \frac{1}{2}$ (either) or $16x+4=2$	A1		
	$x = -\frac{1}{8}$ or equiv	A1	3	CSO

5.

(a)	(Range of f) $\geqslant 0$	B1	1	
(b)(i)	$\text{fg}(x)=\dfrac{1}{(x+2)^2}$	B1	1	OE Maybe in part (ii)
(ii)	$\dfrac{1}{(x+2)^2}=4$			
	$(x+2)^2=\dfrac{1}{4}$	M1		Or $4(x+2)^2=1$
	$x+2=(\pm)\dfrac{1}{2}$	M1		$(2x+5)(2x+3)=0$
	$x=-\dfrac{5}{2},-\dfrac{3}{2}$	A1 A1	4	
(c)(i)	Not one to one	E1	1	OE
(ii)	$x=\dfrac{1}{y+2}$	M1		$x \Leftrightarrow y$
	$y+2=\dfrac{1}{x}$	M1		Attempt to isolate
	$y=\dfrac{1}{x}-2 \quad \left(\dfrac{1-2x}{x}\right)$	A1	3	

6.

	Solution	Marks	Total	Comments
(a)	$[f(x)]$ not $1-1$	E1	1	OE
(b)	$y=\frac{1}{2x+1}$			
	$x=\frac{1}{2y+1}$	M1		swap x and y } either order
	$2y+1=\frac{1}{x}$	M1		a correct next line }
	$[g^{-1}(x)=]\frac{1}{2}\left(\frac{1}{x}-1\right)$ OE	A1	3	$[y=]\frac{1}{2}\left(\frac{1}{x}-1\right)$
(c)	$[g^{-1}(x)]\neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left(\frac{1}{2x+1}\right)^2=\frac{1}{2x+1}$	B1		sight of $\left(\frac{1}{2x+1}\right)^2$ or $\frac{1}{(2x+1)^2}$
	$(2x+1)=(2x+1)^2$ or $2x+1=4x^2+4x+1$ or $\frac{1}{2x+1}=1$ or $2x+1=1$	M1		one correct step, must be one of these four lines
	$x=0$	A1	3	CSO

7.

(a) $y \geq 3$

(b) $y=3+\sqrt{x+2} \Rightarrow y-3=\sqrt{x+2} \Rightarrow x=(y-3)^2-2$

$\Rightarrow g^{-1}(x)=(x-3)^2-2,$ with $x \gg 3$

(c) $g(x)=x \Rightarrow 3+\sqrt{x+2}=x$

$\Rightarrow x+2=(x-3)^2 \Rightarrow x^2-7x+7=0$

$\Rightarrow x=\frac{7\pm\sqrt{21}}{2} \Rightarrow x=\frac{7+\sqrt{21}}{2}$ only

(d) $a=\frac{7+\sqrt{21}}{2}$

8.

(a)	$f(x) \leq 3$	M1A1	2	M1 for $f < 3$, $x \leq 3$ Condone y, f, range
(b)(i)	$y = \frac{2}{x+1}$			
	$x + 1 = \frac{2}{y}$	M1		Attempt to obtain x as a function of y or y as a function of x
	$x = \frac{2}{y} - 1$	M1		$x \leftrightarrow y$ at any stage
	$y / g^{-1}(x) = \frac{2}{x} - 1 = \frac{2-x}{x}$	A1	3	Any correct form
(ii)	$(g^{-1}(x)) \neq -1$	B1	1	
(c)(i)	$h(x) = \frac{2}{3 - x^2 + 1}$	M1		
	$= \frac{2}{4 - x^2} = \frac{2}{(2-x)(2+x)}$	A1	2	
(ii)	$(x \in \mathbb{R})$, $x \neq +2$, $x \neq -2$	B1	1	Condone omit 'x is real' Allow $x^2 \neq 4$

9.

Q	Solution	Marks	Total	Comments
(a)	$f(x)>-3$	M1		'>-3', '$x>-3$' or '$f(x)\geq -3$'
		A1	2	Allow $y>-3$
(b)(i)	$y=e^{2x}-3$ $y+3=e^{2x}$			
	$\ln(y+3)=2x$	M1		swap x and y
		M1		attempt to isolate: $\ln(y\pm A)=Bx$ or reverse
	$(f^{-1}(x))=\frac{1}{2}\ln(x+3)$	A1	3	OE with no further incorrect working Condone $y=$ …..
	Alternative $x \to \times 2 \to e \to -3$ $\div 2 \leftarrow \ln \leftarrow +3 \leftarrow x$ (M1) (M1)			
	$y=\frac{\ln(x+3)}{2}$	(A1)		
(ii)	$x+3=1$	M1		for putting their $p(x)=1$ from $k\ln(p(x))$ in their part (b)(i)
	$x=-2$	A1	2	CSO SC: B2 $x=-2$ with no working, if full marks gained in part (b)(i)
(c)(i)	$(gf(x)=)\ \frac{1}{3(e^{2x}-3)+4}$ $(=)\ \frac{1}{3e^{2x}-5}$ } either OE	B1	1	substituting f into g ISW
(ii)	$\frac{1}{3e^{2x}-5}=1$			
	$1=3e^{2x}-5$ $e^{2x}=2$ OE	M1		Correct removal of their fraction
	$2x=\ln 2$	m1		Correct use of logs leading to $kx=\ln\frac{a}{b}$
	$x=\frac{1}{2}\ln 2$ OE	A1	3	CSO No ISW except for numerical evaluation

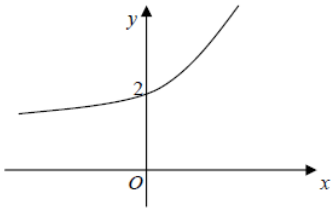
10.

Q	Solution	Marks	Total	Comments
(a)	$f(1)=21$ $f(16)=1$	M1		sight of 1 and 21
	$1 \le f(x) \le 21$	A1	2	allow f(x) replaced by f, y
(b)(i)	$y=\frac{63}{4x-1}$			
	$x=\frac{63}{4y-1}$	M1		reverse x, y } Either order
	$x(4y-1)=63$ or better	M1		one correct step } Either order
	$f^{-1}(x)=\frac{1}{4}\left(\frac{63}{x}+1\right)$ OE	A1	3	condone $y=$
(ii)	$\frac{1}{4}\left(\frac{63}{x}+1\right)=1$			
	$\frac{63}{x}+1=4$, or better	M1		one correct step from their (b)(i) = 1, or x = f(1)
	$(x=)\ 21$	A1	2	note: 21 scores 2/2
(c)(i)	$(fg(x)=)\ \frac{63}{4x^2-1}$	B1	1	
(ii)	$\frac{63}{4x^2-1}=1$			
	$4x^2-1=63$ or better	M1		one correct step from their (c)(i) = 1
	$x^2=16$ OE	A1		eg $(2x+8)(2x-8)=0$, or $x=\pm 4$
	$x=-4$ **ONLY**	A1	3	

11.

Q	Solution	Marks	Total	Comments
(a)	$f(x) \geq 0$	M1 A1	2	$f(x) > 0,\ f \geq 0,\ x \geq 0,\ y > 0,$ range ≥ 0 Condone $y \geq 0$
(b)(i)	$fg(x) = \sqrt{2\left(\frac{10}{x}\right) - 5}$ $\left(= \sqrt{\frac{20}{x} - 5}\right)$ OE	B1	1	No ISW
(ii)	$\sqrt{\frac{20}{x} - 5} = 5$ $\frac{20}{x} = 5^2 + 5$ $x = \frac{2}{3}$	M1 A1	2	Correctly squaring their fg(x) and correctly isolating their x term No ISW
(c)(i)	$y = \sqrt{2x - 5}$ $\left(f^{-1}(x) =\right)\ \frac{x^2 + 5}{2}$	M1 M1 A1	3	Swap x and y Correctly squaring } either order
(ii)	$x^2 = 9$ or if $\sqrt{9}$ or 3 seen $x = 3$ and $x = -3$ rejected	M1 A1	2	Candidate must have scored full marks in (c)(i) (ie no follow through) Must see both

12.

Q	Solution	Marks	Total	Comments
(a)(i)	$f(x) = \ln(2x-3)$ $2x-3 = e^y$ $2y-3 = e^x$	M1 M1		Either order: M1 for antilog M1 for replacing f(x) or y with x
	$(f^{-1}(x) =)\ \frac{1}{2}(e^x+3)$ OE	A1	3	Correct expression in x
(ii)	$f^{-1}(x) > \frac{3}{2}$	B1	1	Do **not** condone $f^{-1}(x) \geq \frac{3}{2}$, $y > \frac{3}{2}$, $x > \frac{3}{2}$ range $> \frac{3}{2}$, $f^{-1} > \frac{3}{2}$
(iii)		M1 A1	2	Correct shape crossing y-axis and above x-axis 2 marked on the y-axis
(b)(i)	$(gf(x) =)\ e^{2\ln(2x-3)} - 4$	M1		Correct composition
	$= e^{\ln(2x-3)^2} - 4$	m1		PI by correct expression
	$= (2x-3)^2 - 4$	A1	3	
(ii)	$(fg(x) =)\ \ln(2(e^{2x}-4)-3)$	M1		OE correct composition
	$\ln(2e^{2x}-11) = \ln 5$ $2e^{2x}-11 = 5$ OE	A1		Correct antilog of correct equation
	$e^{2x} = 8$ $2x = \ln 8$ $x = \frac{1}{2}\ln 8$	A1	3	OE **exact solution**, e.g. $\ln\sqrt{8}$ or $\frac{3}{2}\ln 2$ or $\ln 2^{\frac{3}{2}}$