Revision – Probability (Year 12)

1.

(i) 20% of people in the large town of Carnley support the Residents' Party. 12 people from Carnley are selected at random. Out of these 12 people, the number who support the Residents' Party is denoted by U.

Find

(a) $P(U \leqslant 5)$, [2]

(b) $P(U \geqslant 3)$. [3]

(ii) 30% of people in Carnley support the Commerce Party. 15 people from Carnley are selected at random. Out of these 15 people, the number who support the Commerce Party is denoted by V.

Find $P(V = 4)$. [3]

2.

A random variable X has the distribution $B(8, 0.55)$. Find

(a) $P(X < 7)$, [1]

(b) $P(X = 5)$, [2]

(c) $P(3 \leqslant X < 6)$. [3]

3.

Past experience has shown that when seeds of a certain type are planted, on average 90% will germinate. A gardener plants 10 of these seeds in a tray and waits to see how many will germinate.

(i) Name an appropriate distribution with which to model the number of seeds that germinate, giving the value(s) of any parameters. State any assumption(s) needed for the model to be valid. [4]

(ii) Use your model to find the probability that fewer than 8 seeds germinate. [2]

Later the gardener plants 20 trays of seeds, with 10 seeds in each tray.

(iii) Calculate the probability that there are at least 19 trays in each of which at least 8 seeds germinate. [4]

4.

At a factory that makes crockery the quality control department has found that 10% of plates have minor faults. These are classed as 'seconds'. Plates are stored in batches of 12. The number of seconds in a batch is denoted by X.

(i) State an appropriate distribution with which to model X. Give the value(s) of any parameter(s) and state any assumptions required for the model to be valid. **[4]**

Assume now that your model is valid.

(ii) Find

(a) $P(X = 3)$, **[2]**

(b) $P(X \geqslant 1)$. **[2]**

(iii) A random sample of 4 batches is selected. Find the probability that the number of these batches that contain at least 1 second is fewer than 3. **[4]**

5.

30% of packets of Natural Crunch Crisps contain a free gift. Jan buys 5 packets each week.

(i) The number of free gifts that Jan receives in a week is denoted by X. Name a suitable probability distribution with which to model X, giving the value(s) of any parameter(s). State any assumption(s) necessary for the distribution to be a valid model. **[4]**

Assume now that your model is valid.

(ii) Find

(a) $P(X \leqslant 2)$, **[1]**

(b) $P(X = 2)$. **[2]**

(iii) Find the probability that, in the next 7 weeks, there are exactly 3 weeks in which Jan receives exactly 2 free gifts. **[3]**

6.

It is known that, on average, one match box in 10 contains fewer than 42 matches. Eight boxes are selected, and the number of boxes that contain fewer than 42 matches is denoted by Y.

(i) State two conditions needed to model Y by a binomial distribution. [2]

Assume now that a binomial model is valid.

(ii) Find

(a) $P(Y = 0)$, [2]

(b) $P(Y \geqslant 2)$. [2]

(iii) On Wednesday 8 boxes are selected, and on Thursday another 8 boxes are selected. Find the probability that on one of these days the number of boxes containing fewer than 42 matches is 0, and that on the other day the number is 2 or more. [3]

7.

(i) A random variable, X, has the distribution B(12, 0.85). Find

(a) $P(X > 10)$, **[2]**

(b) $P(X = 10)$, **[2]**

(ii) A random variable, Y, has the distribution B$(2, \frac{1}{4})$. Two independent values of Y are found. Find the probability that the sum of these two values is 1. **[4]**

8.

A random variable X has the distribution B(13, 0.12).

(i) Find $P(X < 2)$. **[3]**

Two independent values of X are found.

(ii) Find the probability that exactly one of these values is equal to 2. **[3]**

9.

(Skip part (c) in Year 12. It will be taught in Year 13.)

For the events A and B,

$$P(A \cap B') = 0.32,\ P(A' \cap B) = 0.11 \text{ and } P(A \cup B) = 0.65.$$

(*a*) Draw a Venn diagram to illustrate the complete sample space for the events A and B. **(3)**

(*b*) Write down the value of $P(A)$ and the value of $P(B)$. **(3)**

(*c*) Find $P(A \mid B')$. **(2)**

(*d*) Determine whether or not A and B are independent. **(3)**

10.

A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take regular exercise. When questioned about their eating habits $\frac{2}{3}$ said they always eat breakfast and, of those who always eat breakfast $\frac{9}{25}$ also took regular exercise.

Find the probability that a randomly selected member of the group

(a) always eats breakfast and takes regular exercise,

(2)

(b) does not always eat breakfast and does not take regular exercise.

(4)

(c) Determine, giving your reason, whether or not always eating breakfast and taking regular exercise are statistically independent.

(2)

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11. **(Skip part (c) in Year 12. It will be taught in Year 13.)**

Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school
K = the event that Kamil is late for school

$P(J) = 0.25$, $P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

(1)

(b) Kamil is late for school.

(2)

Given that Jake is late for school,

(c) find the probability that Kamil is late.

(3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

(2)

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

(1)
