Mathematics

Revision – Probability (Year 12) – Mark Scheme

1.

ia	Binomial stated or implied 0.9806	B1 B1 2	by use of tables or $0.2^a \times 0.8^b$, $a+b=12$
b	0.5583 seen 1 – 0.5583 = 0.442 (3 sfs)	M1 M1 A1 3	add 10 corr terms or 1-(add 3 corr terms): M2 or 1– 0.7946 or 0.205 or 1-0.6774 or 0.323 or 1-0.3907 or 0.609 or add 9 terms or 1-(add 2 or 4 terms): M1
ii	$^{15}C_4 \times 0.3^4 \times 0.7^{11}$ = 0.219 (3 sfs)	M2 A1 3	$^{15}C_4 \times 0.3^{11} \times 0.7^4$: M1

2.

(a)	0.9368 or 0.937	B1 1	
(b)	0.7799 – 0.5230 or $^8C_5 \times 0.45^3 \times 0.55^5$ = 0.2569 or 0.2568 or 0.257	M1 A1 2	Allow 0.9368 – 0.7799
(c)	0.7799 seen – 0.0885 (not 1 – 0.0885) = 0.691 (3 sfs)	M1 M1 A1 3	$^8C_5x0.45^3x0.55^5+^8C_4x0.45^4x0.55^4+^8C_3x0.45^5 \times 0.55^3$: M2 1 term omitted or wrong or extra: M1

3.

(i)	Binomial $n = 10, p = 0.9$ Each seed equally likely germ or P(germ) same for all seeds oe Seeds independent oe	B1 B1 B1 B1 4	Both requ'd. Ignore $q = 0.1$ or seeds grown in same conditions Context nec'y for each B1
(ii)	0.0702 (3 sfs)	B2 2	0.07 or 0.2639: B1 Σ or 1-Σ: 1 term extra or omit or wrong: M1
(iii)	1 – "0.0702" $0.9298^{20} + {}^{20}C_1 \times 0.0702 \times 0.9298^{19}$ = 0.585 (3 sfs)	M1 M1M1 A1 4	Or 0.9298 or 0.93(0) seen M1 each term cao eg ft (ii) 0.2639 → (iii) 0.0178 from correct wking: M3A0 $0.0702^{20}+{}^{20}C_1x0.9298x0.0702^{19}$ ($=2.25\times10^{-21}$): SC M1M1 NB ft (ii) for all M mks. But if 0.1, 0.9 used, must be clear using (ii) rounded

4.

(i)	Binomial $n = 12, p = 0.1$ Plates (or seconds) independent oe Prob of fault same for each plate oe	B1 B1 B1 B1 4	 B(12, 0.1) : B2 NOT: batches indep Comments must be in context Ignore incorrect or irrelevant
(ii)(a)	$0.9744 - 0.8891$ or ${}^{12}C_3 \times 0.9^9 \times 0.1^3$ $= 0.0852$ or 0.0853 (3 sfs)	M1 A1 2	
(b)	$1 - 0.2824$ or $1 - 0.9^{12}$ $=0.718$ (3 sfs)	M1 A1 2	allow $1 - 0.6590$ or $1 - 0.9^{11}$
(iii)	"0.718" and 1 – "0.718" used $(1-0.718)^4 + 4(1-0.718)^3 \times 0.718$ $+ {}^4C_2(1-0.718)^2 \times 0.718^2$ = 0.317 (3 sfs)	B1 M2 A1 4	ft (b) for B1M1M1 M1 for any one term correct (eg opp tail or no coeffs) 1 – P(3 or 4) follow similar scheme M2 or M1 1 – correct wking (= 0.623) B1M2 cao

5.

			Consistent use of $\frac{1}{3}$ or MR of 30% (eg 0.2): ("Consistent" as in Qu 2) (i) B1B0B1B1 (iia) B0 (iib) 0.7901–0.4609 or ${}^5C_2(\frac{2}{3})^3(\frac{1}{3})^2$ M1; = 0.329 (3 sf) A1 (iii) p = "0.3292" M1; ${}^7C_3(1 - \text{"0.3292"})^4(\text{"0.3292"})^3$ M1; = 0.253 (3 sf) A1 ie max 8/10	
i	Binomial or B (5, 0.3) Prob of gift same for all pkts Whether pkt contains gift is indep of other pkts	B1 B1 B1 B1 4	 Prob of gift is constant or fixed or consistent or same oe Obtaining a gift is indep Each time receive a gift is indep Context needed for 3rd & 4th B-mks	Allow mis-spellings but NOT "Biometric" Condone B~(5, 0.3) or B(0.3, 5): B1B1 but B(X = 0.3, n = 5): B1B0 NOT: prob of success const; NOT prob stays same each go One box doesn't affect another. Pkts indep. Gifts indep She buys packets separately Prob of a gift is indep Prob of gift indep of one another & const: B1B1 NOT: Each week is indep NOT: Number of gifts received is indep NOT: Events indep If Geo(0.3) stated, can score max B0B0B1B1 If Geo(5, 0.3) stated, can score max B0B1B1B1
iia	0.8369	B1 1	or 0.837	
b	$0.8369 - 0.5282$ or ${}^5C_2(0.7)^3(0.3)^2$ = 0.3087 or 0.309 (3 sf)	M1 A1 2		
iii	p = "0.3087" ${}^7C_3(1 - \text{"0.3087"})^4(\text{"0.3087"})^3$ = 0.235 (3 sf)	M1 M1 A1 3	(iib) used in a calc'n eg "0.3087" × 3	or B(7, "0.3087") stated or 1 – "0.3087" used instead of "0.3087" n = 35 or 15: max M1M0A0

6.

(i)	Boxes are independent Probability same for each box	B1 B1 2	Both must be in context
(ii)	(a) B(8, 0.1) 0.4305 (b) $1 - P(\leq 1)$ 0.1869	M1 A1 M1 A1 4	B(8, 0.1) stated or 0.1, 0.9 seen and sum of powers =8 0.43[05] correct $1 - 0.8131$ or $1 - (0.9^8 + 8x0.9^7x\ 0.1)$ correct Answer, a.r.t. 0.187
(iii)	$2 \times 0.4305 \times 0.1869$ 0.16092	M1 M1 A1 3	(a) x (b) } 2 x (a) × (b) } Answer, a.r.t. 0.161

7.

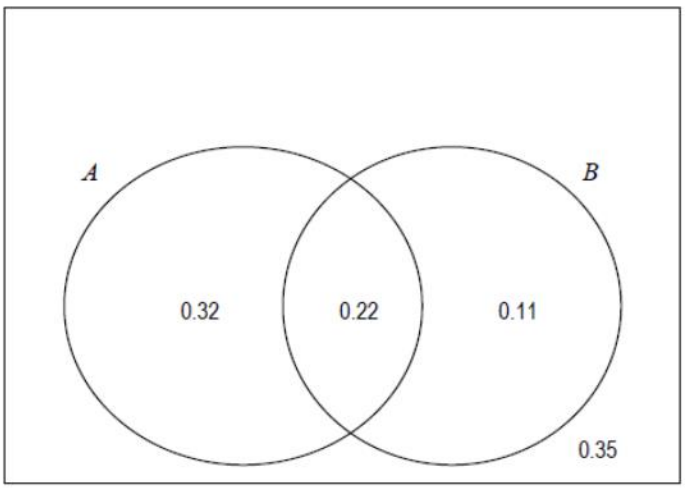
ia	$(1-0.5565)$ or $12\times0.85^{11}\times(1-0.85)+0.85^{12}$ $=0.4435$ or 0.443 or 0.444 (3 sf)	M1 A1 2	or $1-((1-0.85)^{12}\ldots{}^{12}C_{10}\times0.85^{10}(1-0.85)^{2})$ ie $1-$ (all 11 correct binomial terms)	or $1-0.557$ NB $1-0.4435$ (oe): M0A0
b	$0.5565-0.2642$ or ${}^{12}C_{10}(1-0.85)^{2}(0.85)^{10}$ $=0.2923$ or 0.2924 or 0.292 (3 sf)	M1 A1 2		or $0.557-0.264$
ii	$(\frac{3}{4})^2$ AND $\frac{3}{4}\times\frac{1}{4}$ seen (possibly $\times 2$) $(\frac{3}{4})^2\times2\times\frac{3}{4}\times\frac{1}{4}$ oe or $\frac{27}{128}$ or 0.211 $2\times(\frac{3}{4})^2\times2\times\frac{3}{4}\times\frac{1}{4}$ oe $=\frac{27}{64}$ or 0.422 (3 sfs)	M1 M1 M1 A1 4	eg $(\frac{3}{4})^2+\frac{3}{4}\times\frac{1}{4}$ or $2\times(\frac{3}{4})^2+2\times\frac{3}{4}\times\frac{1}{4}$ or $0.5625+0.1875$ or $0.5625+0.375$ or eg 0.5625×0.375 Fully correct method	or $\frac{9}{16}$ and $\frac{3}{16}$ or $\frac{9}{16}$ and $\frac{3}{8}$ eg in table or list Allow even if further incorrect wking Ans 0.211: check wking but probably gets M1M1M0A0 Use of 0.85 instead of $\frac{1}{4}$: MR max M1M1M1A0

8.

(i)		$(1-0.12)^{13}$ or $13\times(1-0.12)^{12}\times0.12$ $(1-0.12)^{13}+13\times(1-0.12)^{12}\times0.12$ $=0.526$ (3 sf)	M1 M1 A1[3]	Either seen Fully correct method	1 – correct terms: M1M0A0
(ii)		${}^{13}C_2\times0.12^2\times(1-0.12)^{11}$ $2\times$ "0.275275" $\times(1-$ "0.275275") $=0.399$ (3 sf)	M1 M1 A1 [3]	or $0.275(\ldots)$ Correct method except allow omit "2 ×"	Allow if × or + something NB unlike 2nd M1 in (i) which is for fully correct method NB $2\times0.12\times0.88$: M0M0A0

9.

(a)



ε

Venn Diagram M1
0.32,0.11 & A,B A1
0.22,0.35 & box A1
(3)

(b) $P(A)=0.32+0.22=0.54;P(B)=0.33$ M1A1ft;A1ft (3)

(c) $P(A|B')=\dfrac{P(A\cap B')}{P(B')}=\dfrac{32}{67}$ awrt 0.478 M1A1 (2)

(d) For independence $P(A\cap B)=P(A)P(B)$
For these data $0.22\neq0.54\times0.33=0.1782$
(OR $P(A|B')\neq P(A)$ for M1A1ft OR $\frac{2}{3}=P(A|B)\neq P(A)=0.54$ for M1A1ft)
$\therefore$ NOT independent
M1A1ft A1ft (3)

10.

(a)	E = take regular exercise B = always eat breakfast $\mathrm{P}(E \cap B) = \mathrm{P}(E \mid B) \times \mathrm{P}(B)$ $= \frac{9}{25} \times \frac{2}{3} = 0.24$ or $\frac{6}{25}$ or $\frac{18}{75}$	M1 A1	(2)
(b)	$\mathrm{P}(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25} = \frac{62}{75}$ or $\mathrm{P}(E' \mid B') = \frac{13}{25}$ or $\mathrm{P}(B' \cap E) = \frac{12}{75}$ or $\mathrm{P}(B \cap E') = \frac{32}{75}$ $\mathrm{P}(E' \cap B') = 1 - \mathrm{P}(E \cup B) = \frac{13}{75}$ or $0.17\dot{3}$	M1 A1 M1 A1	(4)
(c)	$\mathrm{P}(E \mid B) = 0.36 \neq 0.40 = \mathrm{P}(E)$ or $\mathrm{P}(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = \mathrm{P}(E) \times \mathrm{P}(B)$ So E and B are <u>not</u> statistically independent	M1 A1	(2) **[8]**

11.

(a)	$\mathrm{P}(J \cup K) = 1 - 0.7$ or $0.1 + 0.15 + 0.05 =$ <u>0.3</u>	B1	(1)
(b)	$\mathrm{P}(K) = 0.05 + 0.15$ or "0.3" $- 0.25 + 0.15$ or "0.3" $= 0.25 + \mathrm{P}(K) - 0.15$ May be seen on Venn diagram = <u>0.2</u>	M1 A1	(2)
(c)	$[\mathrm{P}(K \mid J)] = \frac{\mathrm{P}(K \cap J)}{\mathrm{P}(J)}$ $= \frac{0.15}{0.25}$ $= \underline{\frac{3}{5}}$ <u>or 0.6</u>	M1 A1 A1	(3)
(d)	$\mathrm{P}(J) \times \mathrm{P}(K) = 0.25 \times 0.2 (= 0.05)$, $\mathrm{P}(J \cap K) = 0.15$ <u>or</u> $\mathrm{P}(K \mid J) = 0.6$, $\mathrm{P}(K) = 0.2$ <u>or</u> may see $\mathrm{P}(J \mid K) = 0.75$ and $\mathrm{P}(J) = 0.25$ not equal therefore not independent	M1 A1ft	(2)
(e)	Not independent so confirms the teacher's suspicion <u>or</u> they are linked (This requires a statement about independence in (d) or in (e))	B1ft	(1)
