Revision – Matrices (Core Pure 1)

1.

A system of three equations is defined by

$$\begin{aligned} kx + 3y - z &= 3 \\ 3x - y + z &= -k \\ -16x - ky - kz &= k \end{aligned}$$

where k is a positive constant.

Given that there is no unique solution to all three equations,

(a) show that $k = 2$

(2)

Using $k = 2$

(b) determine whether the three equations are consistent, justifying your answer.

(3)

(c) Interpret the answer to part (b) geometrically.

(1)

2.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 4 \\ k & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} k-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$$

where k is a constant.

(a) Determine, in simplest form in terms of k, the matrix $\mathbf{MN}$.

(2)

(b) Given that $k = 5$

(i) write down $\mathbf{MN}$

(ii) hence write down $\mathbf{M}^{-1}$

(2)

(c) Solve the simultaneous equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ 5x + 2y - 2z &= 3 \\ 4x + y - 2z &= -1 \end{aligned}$$

(2)

(d) Interpret the answer to part (c) geometrically.

(1)

3.

$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 7 & 2 \\ -5 & 8 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 6 & 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -5 & 2 & 1 \\ 4 & 3 & 8 \\ -6 & 11 & 2 \end{pmatrix}$$

Given that $\mathbf{I}$ is the 3×3 identity matrix,

(a) (i) show that there is an integer k for which

$$\mathbf{AB} - 3\mathbf{C} + k\mathbf{I} = \mathbf{0}$$

stating the value of k

(ii) explain why there can be no constant m such that

$$\mathbf{BA} - 3\mathbf{C} + m\mathbf{I} = \mathbf{0}$$

(4)

(b) (i) Show how the matrix $\mathbf{C}$ can be used to solve the simultaneous equations

$$\begin{aligned} -5x + 2y + z &= -14 \\ 4x + 3y + 8z &= 3 \\ -6x + 11y + 2z &= 7 \end{aligned}$$

(ii) Hence use your calculator to solve these equations.

(3)