Revision – Matrices (Core Pure 1) – Mark Scheme

1.

Question	Scheme	Marks	AOs
(a)	$\begin{vmatrix} k & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -k & -k \end{vmatrix} = k(k+k)-3(-3k+16)-1(-3k-16)$	M1	2.1
	Solves det $= 0 \Rightarrow 2k^2+12k-32=0$ or $k^2+6k-16=0$ To achieve $k=2$ ($k=-8$ must be rejected)	A1	1.1b
		(2)	
	Special case $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -2 & -2 \end{vmatrix} = 2(2+2)-3(-3\times2+16)-1(-3\times2-16)$ Shows det = 0, therefore when $k=2$ there is no unique solution	M1 A0	2.1 1.1b
(b)	Eliminates z to achieve **two** equations in x and y e.g. $5x+2y=1$, $-10x-4y=-2$, $20x+8y=4$ / Eliminates x to achieve **two** equations in y and z e.g. $11y-5z=13$, $22y-10z=26$, $-22y-10z=-26$ / Eliminates y to achieve **two** equations in x and z e.g. $11x+2z=-3$, $22x+4z=-6$, $-44x-8z=12$	M1 A1	3.1a 1.1b
	Must give a **reason:** e.g. Two equations are a linear multiple of each other e.g. shows they are the same equation therefore the equations are **consistent**.	A1	2.4
	Alternative Eliminates two different variables to form two equations, should be one equation from two of the three sections in the main scheme. e.g $5x+2y=1$ and $11y-5z=13$ rearranges and substitutes in to one of the original equations in three variables. e.g. $2x+3\left(\frac{1-5x}{2}\right)-\left(\frac{-3-11x}{2}\right)=3$	M1	3.1a
	Correct equations e.g $5x+2y=1$ and $11y-5z=13$	A1	1.1b
	Shows that the equations are a solution e.g. 3 = 3 therefore **consistent**	A1	2.4
(c)	The three **planes** form a **sheaf.**	B1	2.2a
		(1)	

2.

Question	Scheme	Marks	AOs
(a)	$\mathbf{MN}=\begin{pmatrix} 2k-24 & 0 & 0 \\ k^2-7k+10 & 6k-44 & -10k+50 \\ 4k-20 & 0 & -14 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)(i)	$\mathbf{MN}=\begin{pmatrix} -14 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & -14 \end{pmatrix}$	B1ft	1.1b
(ii)	$\mathbf{M}^{-1}=-\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$	B1	1.1b
		(2)	
(c)	$\mathbf{M}^{-1}=-\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}=\ldots$	M1	1.1b
	$\left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$	A1	1.1b
		(2)	
(d)	The coordinates of the only point at which the **planes** represented by the equations in (c) meet.	B1	2.2a
		(1)	

3.

Question	Scheme	Marks	AOs
(a)	(i) $\mathbf{AB}=\begin{pmatrix}4&-1\\7&2\\-5&8\end{pmatrix}\begin{pmatrix}2&3&2\\-1&6&5\end{pmatrix}=\begin{pmatrix}8+1&12-6&8-5\\14-2&21+12&14+10\\-10-8&-15+48&-10+40\end{pmatrix}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}$	M1	1.1b
	So $\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}-\begin{pmatrix}-15&6&3\\12&9&24\\-18&33&6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$ or $\mathbf{AB}-3\mathbf{C}=\begin{pmatrix}9&6&3\\12&33&24\\-18&33&30\end{pmatrix}+\begin{pmatrix}15&-6&-3\\-12&-9&-24\\18&-33&-6\end{pmatrix}=\begin{pmatrix}24&0&0\\0&24&0\\0&0&24\end{pmatrix}$ and states a value for k	M1	1.1b
	Hence $\mathbf{AB}-3\mathbf{C}-24\mathbf{I}=\mathbf{0}$ so $k=-24$	A1	1.1b
	(ii) Need two things One of: • **BA** is a 2×2 matrix • Finds the matrix **BA** (must be a 2×2 matrix) AND One of: • cannot subtract a 3×3 matrix • finds matrix **3C** and comments that they have different dimensions / can't be done • can't subtract matrices of different sizes • 3**C** or **C** is a 3×3 matrix • **BA** needs to be a 3×3 matrix	B1	2.4
		(4)	
(b)(i)	$\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-14\\3\\7\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-5&2&1\\4&3&8\\-6&11&2\end{pmatrix}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$ Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{C}^{-1}\begin{pmatrix}-14\\3\\7\end{pmatrix}$ Or states $\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}$	M1	1.2
(ii)	$=\frac{1}{360}\begin{pmatrix}-82&7&13\\-56&-4&44\\62&43&-23\end{pmatrix}\begin{pmatrix}-14\\3\\7\end{pmatrix}=\ldots$	M1	1.1b

	$= \begin{pmatrix} -\frac{41}{180} & \frac{7}{360} & \frac{13}{360} \\ -\frac{7}{45} & -\frac{1}{90} & \frac{11}{90} \\ \frac{31}{180} & \frac{43}{360} & -\frac{23}{360} \end{pmatrix} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$ $\mathbf{C}^{-1} \begin{pmatrix} -14 \\ 3 \\ 7 \end{pmatrix} = \ldots$		
	So solution is $x = \frac{7}{2}, y = 3, z = -\frac{5}{2}$ or $(3.5,\ 3,\ -2.5)$	A1	1.1b
		(3)	