Revision – Differentiation (Year 12) - Answers

1.

(a)	$x = 3, \quad y = 9 - 36 + 24 + 3 = 0$	(9 – 36 + 27=0 is OK)	B1 (1)
(b)	$\frac{dy}{dx} = \frac{3}{3}x^2 - 2\times 4\times x + 8 \qquad (x^2 - 8x + 8)$		M1 A1
	When $x = 3$, $\frac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7$		M1
	Equation of tangent: $y - 0 = -7(x - 3)$ $y = -7x + 21$		M1 A1 c.a.o (5)
(c)	$\frac{dy}{dx} = m$ gives $x^2 - 8x + 8 = -7$ $(x^2 - 8x + 15 = 0)$ $(x - 5)(x - 3) = 0$ $x = (3)$ or 5	5	M1 M1 A1
	$\therefore y = \frac{1}{3}5^3 - 4\times 5^2 + 8\times 5 + 3$		M1
	$y = -15\frac{1}{3}$ or $-\frac{46}{3}$		A1 (5)

2.

(a)	$\frac{5-x}{x} = \frac{5}{x} - \frac{x}{x}\left(= \frac{5}{x} - 1\right) \quad (= 5x^{-1} - 1)$		M1
	$\frac{dy}{dx} = 8x, -5x^{-2}$		M1 A1, A1
	When $x = 1$, $\frac{dy}{dx} = 3$	(*)	A1 cso (5)
(b)	At P, $y = 8$		B1
	Equation of tangent: $y - 8 = 3(x - 1)$	$(y = 3x + 5)$ (or equiv.)	M1 A1ft (3)
(c)	Where $y = 0$, $x = -\frac{5}{3} \quad (= k)$	(or exact equiv.)	M1 A1 (2)

3.

(a)	$(3-\sqrt{x})^2 = 9-6\sqrt{x}+x$	M1
	$\div by\sqrt{x} \quad \rightarrow 9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$	A1 c.s.o.
		(2)
(b)	$\int (9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}})dx = \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} - 6x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}(+c)$	M1 A2/1/0
	use $y = \frac{2}{3}$ and $x = 1$: $\quad \frac{2}{3} = 18 - 6 + \frac{2}{3} + c$	M1
	$c = -12$	A1 c.s..o.
	So $\quad y = 18x^{\frac{1}{2}} - 6x + \frac{2}{3}x^{\frac{3}{2}} - 12$	A1f.t.
		(6)

4.

(a)	Evaluate gradient at $x = 1$ to get 4, Grad. of normal $= -\frac{1}{m}$ $\left(= -\frac{1}{4}\right)$	B1, M1	
	Equation of normal: $y - 4 = -\frac{1}{4}(x-1)$ $\quad (4y = -x + 17)$	M1 A1	(4)
(b)	$(3x-1)^2 = 9x^2 - 6x + 1$ (May be seen elsewhere)	B1	
	Integrate: $\frac{9x^3}{3} - \frac{6x^2}{2} + x \ (+C)$	M1 A1ft	
	Substitute (1, 4) to find $c = \ldots$, $\quad c = 3 \quad (y = 3x^3 - 3x^2 + x + 3)$	M1, A1cso	(5)
(c)	Gradient of given line is –2	B1	
	Gradient of (tangent to) C is ≥ 0 (allow >0), so can never equal –2.	B1	(2)

5.

(a) -2 (P), 2 (Q) (± 2 scores B1 B1)	B1, B1	(2)
(b) $y = x^3 - x^2 - 4x + 4$ (May be seen earlier) Multiply out, giving 4 terms	M1	
$\frac{dy}{dx} = 3x^2 - 2x - 4$ (*)	M1 A1cso	(3)
(c) At $x = -1$: $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4 = 1$		
Eqn. of tangent: $y - 6 = 1(x - (-1))$, $y = x + 7$ (*)	M1 A1cso	(2)
(d) $3x^2 - 2x - 4 = 1$ (Equating to "gradient of tangent")	M1	
$3x^2 - 2x - 5 = 0$ $(3x - 5)(x + 1) = 0$ $x = \ldots$	M1	
$x = \frac{5}{3}$ or equiv.	A1	
$y = \left(\frac{5}{3} - 1\right)\left(\frac{25}{9} - 4\right)$, $= \frac{2}{3} \times \left(-\frac{11}{9}\right) = -\frac{22}{27}$ or equiv.	M1, A1	(5)

6.

(a) $\frac{dy}{dx} = 6x^2 - 10x - 4$	M1 A1	(2)
(b) $6x^2 - 10x - 4 = 0$	M1	
$2(3x + 1)(x - 2)$ [=0]	M1	
$x = 2$ or $-\frac{1}{3}$ (both x values)	A1	
Points are $(2, \underline{-10})$ and $(-\frac{1}{3}, 2\frac{19}{27}$ or $\frac{73}{27}$ or 2.70 or better) (both y values)	A1	(4)
(c) $\frac{d^2y}{dx^2} = 12x - 10$	M1 A1	(2)
(d) $x = 2 \Rightarrow \frac{d^2y}{dx^2} (= 14) \underline{> 0} \therefore [(2, -10)]$ is a $\underline{\text{Min}}$	M1	
$x = -\frac{1}{3} \Rightarrow \frac{d^2y}{dx^2} (= -14) \underline{< 0} \therefore [(-\frac{1}{3}, \frac{73}{27})]$ is a $\underline{\text{Max}}$	A1	(2)

7.

(a)	$\frac{dy}{dx} = 3x^2 - 16x + 20$	M1 A1	
	$3x^2 - 16x + 20 = 0$ $(3x - 10)(x - 2) = 0$ $x = \ldots,$ $\frac{10}{3}$ and 2	dM1, A1	(4)
(b)	$\frac{d^2y}{dx^2} = 6x - 16$ At $x = 2$, $\frac{d^2y}{dx^2} = \ldots$	M1	
	-4 (or < 0, or both), therefore maximum	A1ft	(2)
(c)	$\int (x^3 - 8x^2 + 20x)dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2}$ $(+C)$	M1 A1 A1	(3)
(d)	$4 - \frac{64}{3} + 40$ $\left(= \frac{68}{3}\right)$	M1	
	A: $x = 2$: $y = 8 - 32 + 40 = 16$ (Maybe scored elsewhere)	B1	
	Area of Δ = $\frac{1}{2}\left(\frac{10}{3} - 2\right) \times 16$ $\left(\frac{1}{2}(x_B - x_A) \times y_A\right)$ $\left(= \frac{32}{3}\right)$	M1	
	Shaded area $= \frac{68}{3} + \frac{32}{3} = \frac{100}{3}$ $\left(= 33\frac{1}{3}\right)$	M1 A1	(5)

8.

(a) $3x^2 \to cx^3$ or $-6 \to cx$ or $-8x^{-2} \to cx^{-1}$	M1	
$f(x) = \frac{3x^3}{3} - 6x - \frac{8x^{-1}}{-1}$ $(+C)$ $\left(x^3 - 6x + \frac{8}{x}\right)$	A1 A1	
Substitute $x = 2$ and $y = 1$ into a 'changed function' to form an equation in C.	M1	
$1 = 8 - 12 + 4 + C$ $C = 1$	A1cso	(5)
(b) $3 \times 2^2 - 6 - \frac{8}{2^2}$	M1	
$= 4$	A1	
Eqn. of tangent: $y - 1 = 4(x - 2)$	M1	
$y = 4x - 7$ (Must be in this form)	A1	(4)

9.

(a) $f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1}(+c)$ $-\frac{3}{x}$ is OK

M1A1

$(3, 7\frac{1}{2})$ gives $\frac{15}{2} = 9 - \frac{3}{3} + c$ 3^2 or 3^{-1} are OK instead of 9 or $\frac{1}{3}$

M1A1f.t.

$c = -\frac{1}{2}$

A1 (5)

(b) $f(-2) = 4 + \frac{3}{2} - \frac{1}{2}$ (*)

B1c.s.o. (1)

(c) $m = -4 + \frac{3}{4}$, $= -3.25$

M1,A1

Equation of tangent is: $y - 5 = -3.25(x + 2)$

M1

$4y + 13x + 6 = 0$ o.e.

A1 (4)

10.

(a) Perimeter $\Rightarrow 2x + 2y + \pi x = 80$ B1

Area $\rightarrow A = 2xy + \frac{1}{2}\pi x^2$ B1

$y = \frac{80 - 2x - \pi x}{2}$ and sub in to A M1

$\Rightarrow A = 80x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

i.e. $A = 80x - (2 + \frac{\pi}{2})x^2$ * A1 c.s.o (4)

(b) $\frac{dA}{dx} = 80 - 2(2 + \frac{\pi}{2})x$ or $80 - 4x - \pi x$ (or equiv.) M1, A1

$\frac{dA}{dx} = 0 \Rightarrow 40 = (2 + \frac{\pi}{2})x$ so $x = , \frac{40}{2 + \frac{\pi}{2}}$ or $\frac{80}{4 + \pi}$ or Awrt 11.2 M1, A1 (4)

(c) $\frac{d^2A}{dx^2} = -4 - \pi$ M1

< 0 $\therefore A$ is Max A1 (2)

(d) Max Area $= 80(b) - (2 + \frac{\pi}{2})(b)^2$ M1

$= \underline{\underline{448(m^2)}}$ (448 only for A1) A1 cao (2)

11.

(a) $4x^2 + 6xy = 600$			M1 A1
$V = 2x^2y = 2x^2\left(\frac{600 - 4x^2}{6x}\right)$	$V = 200x - \frac{4x^3}{3}$ (*)		M1 A1cso (4)
(b) $\frac{dV}{dx} = 200 - 4x^2$			B1
Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$ (7.07…)			M1 A1
Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943$ cm^3	Allow awrt		M1 A1 (5)
(c) $\frac{d^2V}{dx^2} = -8x$ Negative,	$\therefore$ Maximum		M1, A1ft (2)