Revision – Binomial Expansion (Year 13)

1.

Given that x is small, show that $\left(\frac{4}{2-5x}\right)^3 \approx a + bx + cx^2$, where a, b and c are integers.

Find the range of values of x for which your expansion would be valid.

(6 marks)

2.

(i) Show that $(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$ for small values of x. *(3 marks)*

(ii) Find the range of values of x for which your expansion would be valid.

(1 mark)

(iii)

Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

3.

Hence show that $\sqrt{\frac{2+3x}{8}} \approx a + bx + cx^2$ for small values of x, where a, b and c are constants to be found.

4.

(a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

5.

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures.

(4)

6.

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)

7.

(a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a+bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) the value of a and the value of b,

(5)

(c) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

8.

(i) Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\frac{1+x^2}{(2+x)^2}$. [2]

9.

(i) Expand $(1+ax)^{-4}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) The coefficients of x and x^2 in the expansion of $(1+bx)(1+ax)^{-4}$ are 1 and -2 respectively. Given that $a > 0$, find the values of a and b. [5]

10.

(i) Show that $\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$. [3]
