Answers - Revision – Binomial Expansions (Year 13)

1.

$8+60x+300x^2$

$|x|<\frac{2}{5}$

2.

(i)

$\sqrt[3]{8}\left(1+\frac{3}{8}x\right)^{\frac{1}{3}}$

$=2\left(1+\frac{1}{3}\left(\frac{3}{8}x\right)-\frac{1}{9}\left(\frac{3}{8}x\right)^2\right)$

$=2+\frac{1}{4}x-\frac{1}{32}x^2$

(ii)

| x | < 8/3

(iii)

$x=\frac{1}{3}:\ \sqrt[3]{8+1}=2+\frac{1}{4}\times\frac{1}{3}-\frac{1}{32}\times\left(\frac{1}{3}\right)^2$

$\sqrt[3]{9}=\frac{576+24-1}{288}=\frac{599}{288}$

3.

$\frac{1}{2}+\frac{3}{8}x-\frac{9}{64}x^2$

4.

(a)

$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$

$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets — B1

Expands $(1+**x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})(**x)$; — M1;

$= \frac{1}{2}\left[\ 1+(-\frac{1}{2})(**x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(**x)^2+\ldots\ \right]$

with $**\neq 1$

A correct simplified or an un-simplified $[\ \ldots\ldots\ ]$ expansion with candidate's followed through $(**x)$ — A1√

$= \frac{1}{2}\left[\ 1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+\ldots\ \right]$

Award SC M1 if you see $(-\frac{1}{2})(**x)+\frac{(-\frac{1}{2})(-\frac{1}{2})}{2!}(**x)^2$

$\frac{1}{2}\left[\ 1+\frac{3}{8}x;\ldots\ \right]$

$= \frac{1}{2}\left[\ 1+\frac{3}{8}x;+\frac{27}{128}x^2+\ldots\ \right]$ — A1 isw

SC: $K\left[\ 1+\frac{3}{8}x+\frac{27}{128}x^2+\ldots\ \right]$

$\frac{1}{2}\left[\ \ldots\ldots;\frac{27}{128}x^2\ \right]$ — A1 isw

$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+\ldots\right\}$ — *Ignore subsequent working*

[5]

(b)

$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\ldots\right)$ — Writing $(x+8)$ multiplied by candidate's part (a) expansion. — M1

$= \frac{1}{2}x+\frac{3}{16}x^2+\ldots\ldots$
$+4+\frac{3}{2}x+\frac{27}{32}x^2+\ldots\ldots$

Multiply out brackets to find a constant term, two x terms and two x^2 terms. — M1

$= 4+2x;+\frac{33}{32}x^2+\ldots$

Anything that cancels to $4+2x;\ \frac{33}{32}x^2$ — A1; A1

[4]

5.

(a) $27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$ — Forming this identity — M1

$x=-\frac{2}{3}$, $12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$

$x=1$, $27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$

Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations. — M1

Both $B=4$ and $C=3$ — A1

(Note the A1 is dependent on both method marks in this part.)

Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A$
$\Rightarrow A=0$

$x=0$, $16=2A+B+4C$
$\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$

Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$. — B1

(4)

(b) $f(x)=\frac{4}{(3x+2)^2}+\frac{3}{(1-x)}$

$=4(3x+2)^{-2}+3(1-x)^{-1}$ — Moving powers to top on any one of the two expressions — M1

$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$

$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$

$=1\left\{\underline{1+(-2)\left(\frac{3x}{2}\right);+\frac{(-2)(-3)}{2!}\left(\frac{3x}{2}\right)^2+\ldots}\right\}$

$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$

Either $1\pm(-2)\left(\frac{3x}{2}\right)$ or $1\pm(-1)(-x)$ from either first or second expansions respectively — dM1;

Ignoring 1 and 3, any one correct $\{\ldots\ldots\}$ expansion. — A1

Both $\{\ldots\ldots\}$ correct. — A1

$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$

$=4+0x;+\frac{39}{4}x^2$ — $4+(0x)$; $\frac{39}{4}x^2$ — A1; A1

(6)

(c)

$\text{Actual} = f(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$

$= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$

Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).

M1

Or

$\text{Actual} = f(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$

$= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$

Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.

M1

$\text{Estimate} = f(0.2) = 4 + \frac{39}{4}(0.2)^2$

$= 4 + 0.39 = 4.39$

Attempt to find an estimate for f(0.2) using their answer to (b)

M1√

$\%\text{age error} = \dfrac{|4.39-4.341715976...|}{4.341715976...} \times 100$

$\left|\dfrac{\text{their estimate - actual}}{\text{actual}}\right| \times 100$

M1

$= 1.112095408... = 1.1\%\,(2\text{sf})$

1.1%

A1 cao

(4)

6.

(a)

$A = 2$

B1

$2x^2+5x-10 = A(x-1)(x+2)+B(x+2)+C(x-1)$

$x \to 1 \qquad -3 = 3B \Rightarrow B = -1$

M1 A1

$x \to -2 \qquad -12 = -3C \Rightarrow C = 4$

A1 **(4)**

(b)

$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$

M1

$(1-x)^{-1} = 1 + x + x^2 + \ldots$

B1

$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$

B1

$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$

M1

$= 5 + \ldots$ ft their $A - B + \frac{1}{2}C$

A1 ft

$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $0x$ stated or implied

A1 A1 **(7)**

7.

(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1	
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \ldots$	M1 A1	
	$=1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \ldots$		
	$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots$	M1 A1	**(5)**
(b)	$f(x) = (a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \ldots\right)$		
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0 \quad (3a+b=0)$	M1	
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16} \quad (9a+4b=3)$ A1 either correct	M1 A1	
	Leading to $a=-1,\ b=3$	M1 A1	**(5)**
(c)	Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$	M1 A1ft	
	$=\frac{27}{16}$ cao	A1	**(3)**

8.

(i) $(1+\frac{x}{2})^{-2}$ $=1+(-2)(\frac{x}{2})+\frac{-2.-3}{2}(\frac{x}{2})^2+\frac{-2.-3.-4}{3!}(\frac{x}{2})^3$	M1		Clear indication of method of ≥ 3 terms
$=1-x$	B1		First two terms, not dependent on M1
$+\frac{3}{4}x^2-\frac{1}{2}x^3$	A1		For both third and fourth terms
$(2+x)^{-2} = \frac{1}{4}(\text{their exp of } (1+ax)^{-2})$ mult out	√B1		Correct: $\frac{1}{4}-\frac{1}{4}x+\frac{3}{16}x^2-\frac{1}{8}x^3$
$\lvert x\rvert<2$ or $-2<x<2$ (but not $\lvert\frac{1}{2}x\rvert<1$)	B1	**5**	
(ii) If (i) is $a+bx+cx^2+dx^3$ evaluate $b+d$	M1		
$-\frac{3}{8}\ (x^3)$	√A1	**2**	Follow-through from $b+d$
		7	

9.

(i) $1-4ax+\ldots$	B1		
$\frac{-4.-5}{1.2}(ax)^2$ or $\frac{-4.-5}{1.2}a^2x^2$ or $\frac{-4.-5}{1.2}ax^2$	M1		Do not accept $\binom{-4}{2}$ unless 10 also appears
$\ldots+10a^2x^2$	A1	3	
(ii) f.t. (their cf x) + b(their const cf) = 1	√B1		Expect $b-4a=1$
f.t. (their cf x^2) + b(their cf x) $=-2$	√B1		Expect $10a^2-4ab=-2$
Attempt to eliminate 'b' and produce equation in 'a'	M1		Or eliminate 'a' and produce equation in 'b'
Produce $6a^2+4a=2$ AEF	A1		Or $6b^2+4b=42$ AEF
$a=\frac{1}{3}$ and $b=\frac{7}{3}$ only	A1	5	Made clear to be only (final) answer

10.

(i) $(1-x)^{\frac{1}{2}}=1-\frac{1}{2}x-\frac{1}{8}x^2$ ignoring x^3 etc	**B2**		SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq-\frac{1}{8}$ or 0
$(1+x)^{-\frac{1}{2}}=1-\frac{1}{2}x+\frac{3}{8}x^2$ ignoring x^3 etc	**B2**		SR Allow B1 for $1-\frac{1}{2}x+kx^2$, $k\neq\frac{3}{8}$ or 0
Product = $1-x+\frac{1}{2}x^2$ ignoring x^3 etc	**B1**		**AG**; with (at least) 1 intermediate step (cf x^2)
		5	
(ii) $\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	**B1**		
$\frac{37}{49}$ or $1-\frac{2}{7}+\frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	**B1**		
$\frac{\sqrt{5}}{3}\approx\frac{37}{49}\Rightarrow\sqrt{5}\approx\frac{111}{49}$	**B1**		**AG**
		3	