

Paper Class – Exponentials and Logarithms

1.

In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

- (i) Find the time taken for the mass to decrease to half of its original value. [3]

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

- (ii) Find the time at which both substances are decaying at the same rate. [8]
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2.

A pan of water is heated until it reaches 100°C . Once the water reaches 100°C , the heat is switched off and the temperature $T^\circ\text{C}$ of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

- (i) Write down the value of a . [1]

- (ii) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15°C per minute.

- (iii) Calculate the value of k . [3]

- (iv) Find the time taken for the temperature of the water to drop from 100°C to 45°C . [3]

- (v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100°C . Suggest how the equation for the temperature as the water cools would be modified by this. [1]
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3.

In an experiment 500 fruit flies were released into a controlled environment. After 10 days there were 650 fruit flies present.

Munirah believes that N , the number of fruit flies present at time t days after the fruit flies are released, will increase at the rate of 4.4% per day. She proposes that the situation is modelled by the formula $N = Ak^t$.

- (i) Write down the values of A and k . [2]

- (ii) Determine whether the model is consistent with the value of N at $t = 10$. [2]

- (iii) What does the model suggest about the number of fruit flies in the long run? [1]
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Subsequently it is found that for large values of t the number of fruit flies in the controlled environment oscillates about 750. It is also found that as t increases the oscillations decrease in magnitude.

Munirah proposes a second model in the light of this new information.

$$N = 750 - 250 \times e^{-0.092t}$$

- (iv) Identify three ways in which this second model is consistent with the known data. [3]
- (v) (A) Identify one feature which is not accounted for by the second model. [1]
- (B) Give an example of a mathematical function which needs to be incorporated in the model to account for this feature. [1]

4.

A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n , and the number of months after launch, t , can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

- (i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. [2]
- (ii) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

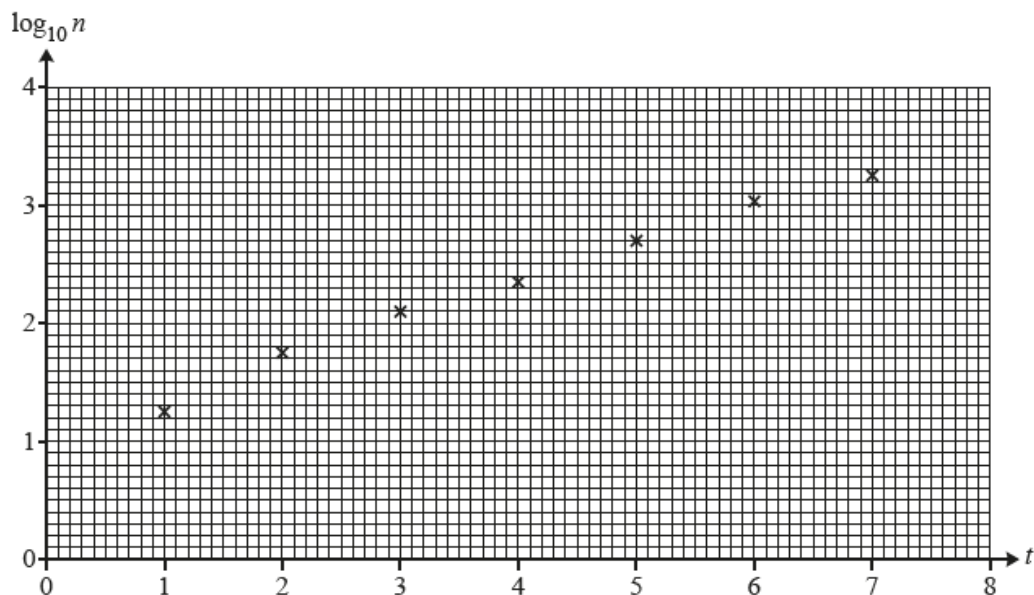


Fig. 5

- Find estimates of the values of a and k . [4]
- (iii) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. [4]
- (iv) Give a reason why the model may not be appropriate for large values of t . [1]

5.

The intensity of the sun's radiation, y watts per square metre, and the average distance from the sun, x astronomical units, are shown in Fig. 11 for the planets Mercury and Jupiter.

	x	y
Mercury	0.3075	14 400
Jupiter	4.950	55.8

Fig. 11

The intensity y is proportional to a power of the distance x .

- (i) Write down an equation for y in terms of x and two constants. [1]
- (ii) Show that the equation can be written in the form $\ln y = a + b \ln x$. [2]
- (iii) In the Printed Answer Booklet, complete the table for $\ln x$ and $\ln y$ correct to 4 significant figures. [2]
- (iv) Use the values from part (iii) to find a and b . [3]
- (v) Hence rewrite your equation from part (i) for y in terms of x , using suitable numerical values for the constants. [2]
- (vi) Sketch a graph of the equation found in part (v). [2]
- (vii) Earth is 1 astronomical unit from the sun. Find the intensity of the sun's radiation for Earth. [1]

From the Printed Answer Booklet:

(iii)

	$\ln x$	$\ln y$
Mercury		
Jupiter		

6.

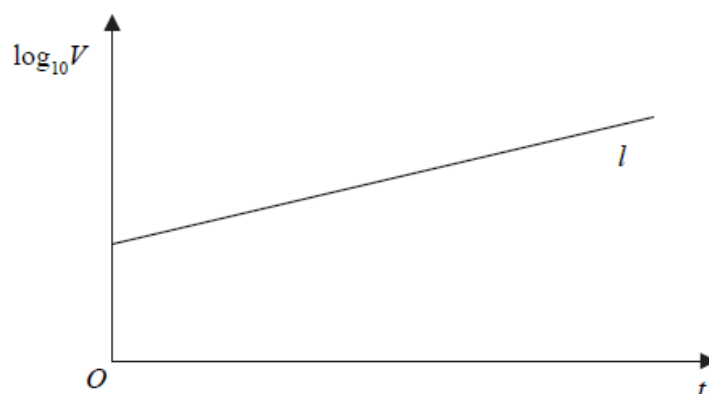


Figure 3

The value of a rare painting, £ V , is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

- (a) Find, to 4 significant figures, the value of p and the value of q . (4)
- (b) With reference to the model interpret
- (i) the value of the constant p ,
 - (ii) the value of the constant q . (2)
- (c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds. (2)
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7.

- (i) (A) Sketch the graph of $y = 3^x$. [1]

- (B) Give the coordinates of any intercepts. [1]

The curve $y = f(x)$ is the reflection of the curve $y = 3^x$ in the line $y = x$.

- (ii) Find $f(x)$. [1]
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8.

In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for t hours. Over the course of the year she realises that her percentage mark for a test, p , may be modelled by the following formula, where A , B and C are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.
- Aaishah had a mark of 22 when she didn't spend any time revising.

- (i) Find the values of A , B and C . [3]

- (ii) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve? [2]

- (iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work? [2]

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p .

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

- (iv) Verify that the data is consistent with the new formula. [2]
- (v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. [2]
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9.

The value, $\pounds V$, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was $\pounds 32\,000$ on 1st January 2005 and $\pounds 50\,000$ on 1st January 2012

- (a) (i) find p to 4 decimal places,
- (ii) show that A is approximately 24800 (4)
- (b) With reference to the model, interpret
- (i) the value of the constant A ,
- (ii) the value of the constant p . (2)

Using the model,

- (c) find the year during which the value of the car first exceeds $\pounds 100\,000$ (4)
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10.

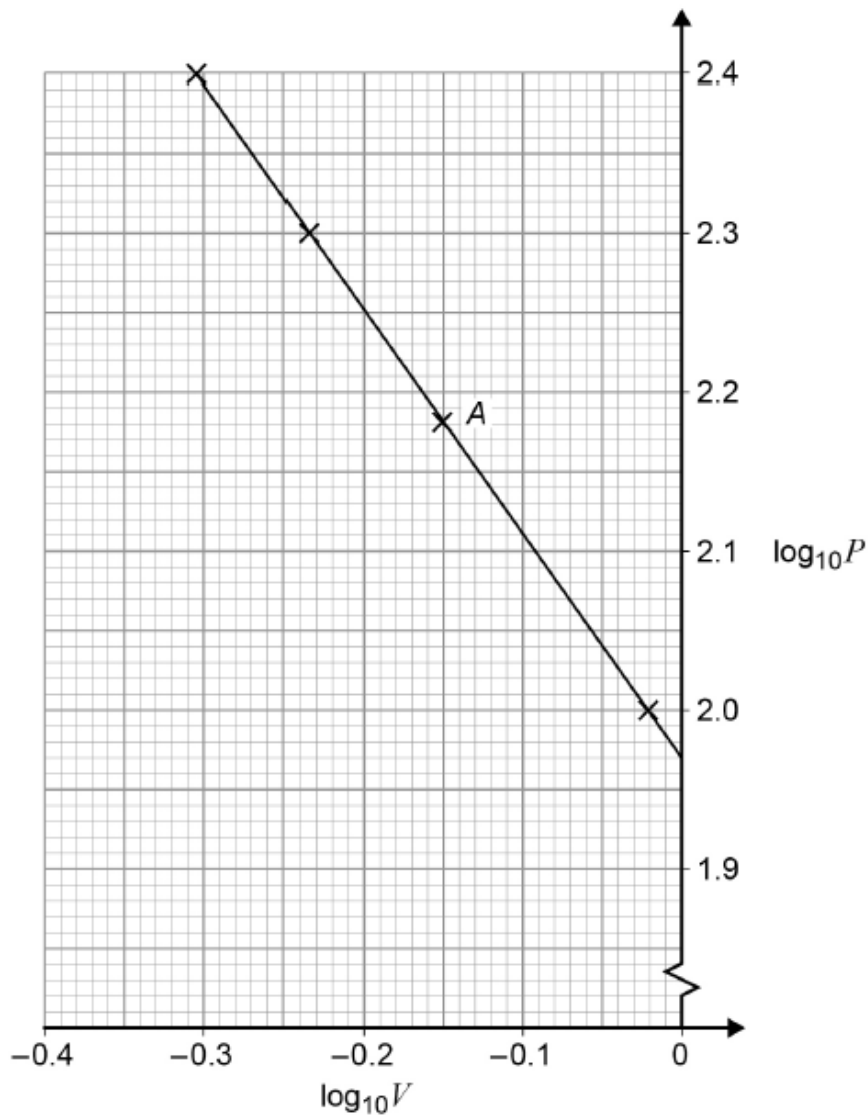
Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



- (a) Find the value of P and the value of V for the data point labelled A on the graph. [2 marks]
- (b) Calculate the value of each of the constants c and d . [4 marks]
- (c) Estimate the pressure of the gas when the volume is 2 litres. [2 marks]