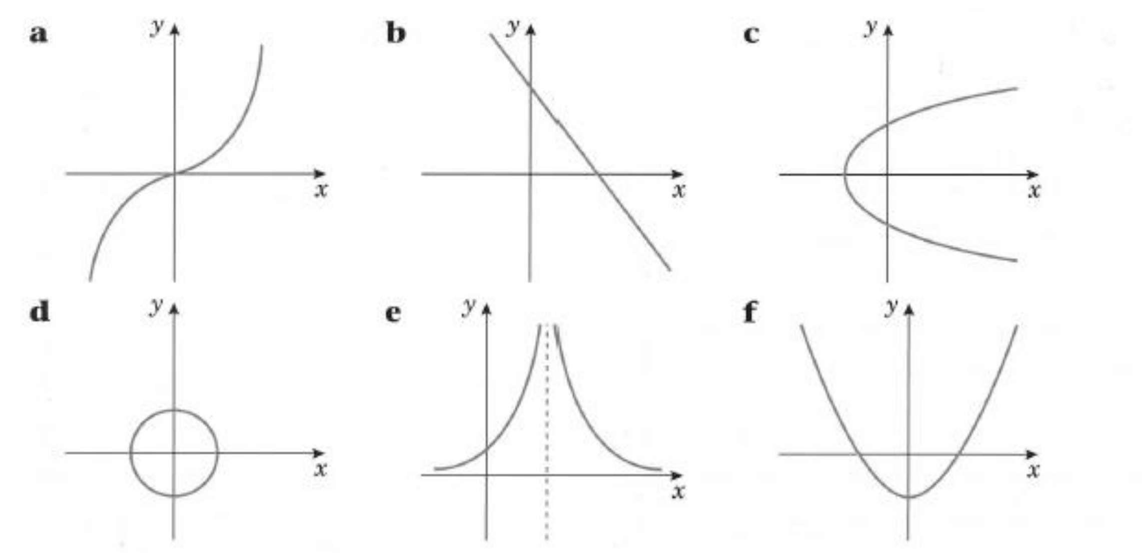
Functions

1. Identify the type of mapping for each of the graphs given below.



2.
The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \ x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $f^{-1}(x)$. *(3 marks)*

3.
Given that

$$f(x) = \frac{4}{3x+5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \qquad x > 0$$

(a) state the range of f,

(2)

(b) find $f^{-1}(x)$,

(3)

(c) find $fg(x)$.

(1)

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.

(4)

4.

The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x+5}, \qquad \text{for real values of } x, \ x \geqslant -2.5$$

$$g(x) = \frac{1}{4x+1}, \qquad \text{for real values of } x, \ x \neq -0.25$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) State the domain of f^{-1}. *(1 mark)*

(c) The composite function fg is denoted by h.

(i) Find an expression for $h(x)$. *(1 mark)*

(ii) Solve the equation $h(x) = 3$. *(3 marks)*

5.

The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 4$. *(4 marks)*

(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

6.

The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x+1} \qquad \text{for real values of } x, \quad x \neq -0.5$$

(a) Explain why f does not have an inverse. *(1 mark)*

(b) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(c) State the range of g^{-1}. *(1 mark)*

(d) Solve the equation $fg(x) = g(x)$. *(3 marks)*

7.

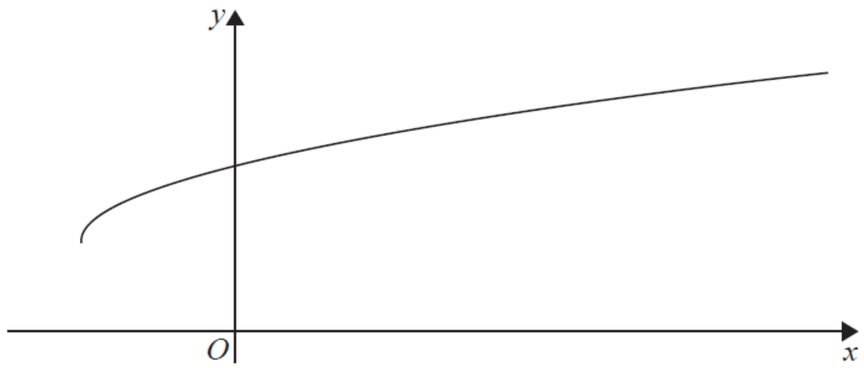


Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \qquad x \geqslant -2$$

(a) State the range of g.

(1)

(b) Find $g^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x for which

$$g(x) = x$$

(4)

(d) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

8.

The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \quad \text{for real values of } x, \ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

9.

The functions f and g are defined with their respective domains by

$$f(x) = e^{2x} - 3, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{3x+4}, \quad \text{for real values of } x, \ x \neq -\frac{4}{3}$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

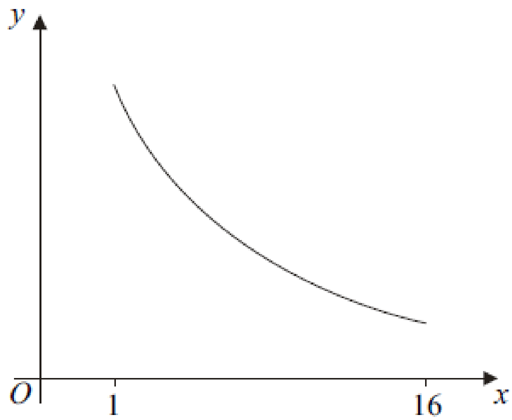
(ii) Solve the equation $f^{-1}(x) = 0$. *(2 marks)*

(c) (i) Find an expression for $gf(x)$. *(1 mark)*

(ii) Solve the equation $gf(x) = 1$, giving your answer in an exact form. *(3 marks)*

10.

The curve with equation $y = \dfrac{63}{4x - 1}$ is sketched below for $1 \leqslant x \leqslant 16$.



The function f is defined by $\mathrm{f}(x) = \dfrac{63}{4x - 1}$ for $1 \leqslant x \leqslant 16$.

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 1$. *(2 marks)*

(c) The function g is defined by $\mathrm{g}(x) = x^2$ for $-4 \leqslant x \leqslant -1$.

(i) Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 1$. *(3 marks)*

11.

The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x - 5}, \quad \text{for } x \geqslant 2.5$$

$$\mathrm{g}(x) = \frac{10}{x}, \quad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 5$. *(2 marks)*

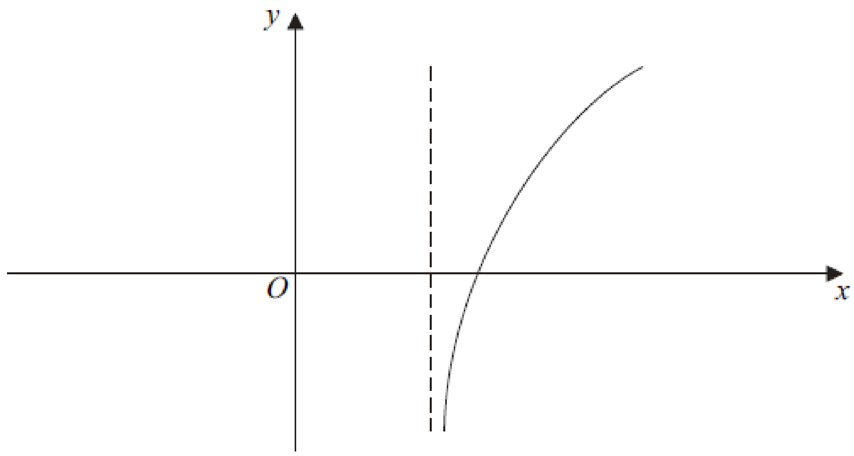
(c) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 7$. *(2 marks)*

12.

The curve with equation $y = f(x)$, where $f(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.



(a) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) State the range of f^{-1}. *(1 mark)*

(iii) Sketch, on the axes given on the opposite page, the curve with equation $y = f^{-1}(x)$, indicating the value of the y-coordinate of the point where the curve intersects the y-axis. *(2 marks)*

(b) The function g is defined by

$$g(x) = e^{2x} - 4, \quad \text{for all real values of } x$$

(i) Find $gf(x)$, giving your answer in the form $(ax - b)^2 - c$, where a, b and c are integers. *(3 marks)*

(ii) Write down an expression for $fg(x)$, and hence find the exact solution of the equation $fg(x) = \ln 5$. *(3 marks)*