Past Paper Questions – Pack 1

1.

The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

2.

The quadratic equation $(k+1)x^2 + 12x + (k-4) = 0$ has real roots.

Find the possible value of k.

(4 marks)

3.

The quadratic equation

$$(2k-3)x^2 + 2x + (k-1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

4.

Find the set of values of x for which

(*a*) $3(2x + 1) > 5 - 2x$,

(2)

(*b*) $2x^2 - 7x + 3 > 0$,

(4)

(*c*) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

5 .

(a) (i) Express $x^2 - 4x + 9$ in the form $(x-p)^2 + q$, where p and q are integers.
(2 marks)

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

6.

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$.
(2)

(*b*) Factorise $2x^3 + x^2 - 25x + 12$ completely.
(4)

7.

The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers.
(2 marks)

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

8.

(*a*) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.
(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(*b*) find the value of p and the value of q.
(4)

9.

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x.$$

(2)

10.

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

11.

(a) The expression $(1-2x)^4$ can be written in the form

$$1+px+qx^2-32x^3+16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3 marks)

(b) Find the coefficient of x in the expansion of $(2+x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1-2x)^4(2+x)^9$. *(3 marks)*

12.

The point A has coordinates $(1, 7)$ and the point B has coordinates $(5, 1)$.

(a) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x+2y=17$. *(2 marks)*

(b) The line AB intersects the line with equation $x-4y=8$ at the point C. Find the coordinates of C. *(3 marks)*

(c) Find an equation of the line through A which is perpendicular to AB. *(3 marks)*

13.

The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively.

(a) (i) Show that the gradient of AB is $-\frac{3}{2}$. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. *(2 marks)*

(b) (i) Find an equation of the line which passes through B and which is perpendicular to the line AB. *(2 marks)*

(ii) The point C has coordinates $(k, 7)$ and angle ABC is a right angle.

Find the value of the constant k. *(2 marks)*

14.

The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

15.

The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B. Find the coordinates of B. *(3 marks)*

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k. *(3 marks)*