

Mixed Exam Questions – Set 12

1.

Tessa owns a small clothes shop in a seaside town. She records the weekly sales figures, £ w , and the average weekly temperature, $t^{\circ}\text{C}$, for 8 weeks during the summer.

The product moment correlation coefficient for these data is -0.915

- (a) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the correlation between sales figures and average weekly temperature is negative. (3)
- (b) Suggest a possible reason for this correlation. (1)

Tessa suggests that a linear regression model could be used to model these data.

- (c) State, giving a reason, whether or not the correlation coefficient is consistent with Tessa's suggestion. (1)
- (d) State, giving a reason, which variable would be the explanatory variable. (1)

Tessa calculated the linear regression equation as $w = 10\,755 - 171t$

- (e) Give an interpretation of the gradient of this regression equation. (1)

2.

[In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O .]

A particle P moves with constant acceleration.

At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\text{ m s}^{-1}$

At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\text{ m}$.

- (a) Show that the magnitude of the acceleration of P is 2.5 m s^{-2} (4)

At the instant when P leaves the point A , the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\text{ m s}^{-2}$

At the instant when P reaches the point B , the direction of motion of P is north east.

- (b) Find the time it takes for P to travel from A to B . (4)

3.

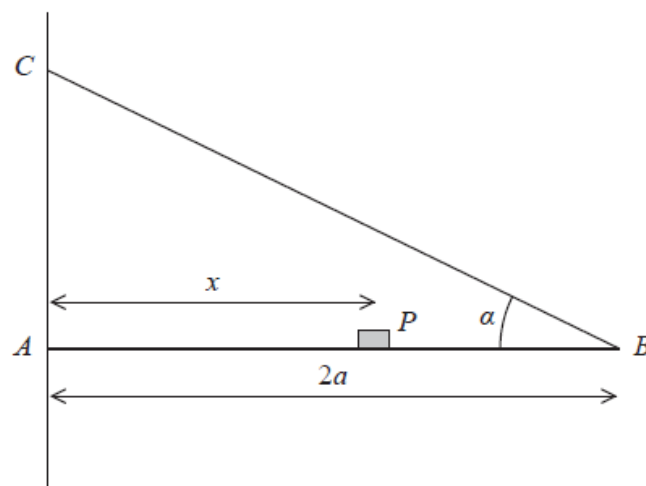


Figure 3

A plank, AB , of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C , which is vertically above A .

A small block of mass $3M$ is placed on the plank at the point P , where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

- (a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$ (3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

- (b) Find x in terms of a . (2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

- (c) Find the value of $\tan \beta$ (5)

The rope will break if the tension in it exceeds $5Mg$.

- (d) Explain how this will restrict the possible positions of P . You must justify your answer carefully. (3)

4.

[In this question position vectors are given relative to a fixed origin O]

At time t seconds, where $t \geq 0$, a particle, P , moves so that its velocity $\mathbf{v} \text{ m s}^{-1}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j}) \text{ m}$.

(a) Find the acceleration of P when $t = 4$ (3)

(b) Find the position vector of P when $t = 4$ (3)

5.

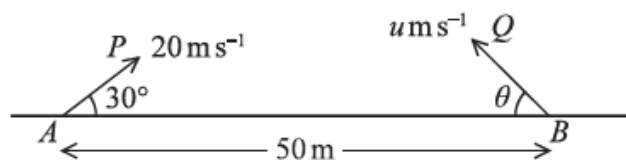


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q , are projected in the vertical plane containing AB .

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB .

Ball Q is projected from B with speed $u \text{ m s}^{-1}$ at angle θ to BA , as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q . (6)

(b) Find (6)

- (i) the size of angle θ ,
- (ii) the value of u .

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers. (1)

6.

In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions east and north respectively.

A particle R of mass 2 kg is moving on a smooth horizontal surface under the action of a single horizontal force $\mathbf{F}\text{ N}$. At time t seconds, the velocity $\mathbf{v}\text{ ms}^{-1}$ of R , relative to a fixed origin O , is given by $\mathbf{v} = (pt^2 - 3t)\mathbf{i} + (8t + q)\mathbf{j}$, where p and q are constants and $p < 0$.

(a) Given that when $t = 0.5$ the magnitude of $\mathbf{F}$ is 20 , find the value of p . [6]

When $t = 0$, R is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})\text{ m}$.

(b) Find, in terms of q , an expression for the displacement vector of R at time t . [4]

When $t = 1$, R is at a point on the line L , where L passes through O and the point with position vector $2\mathbf{i} - 8\mathbf{j}$.

(c) Find the value of q . [3]
