Volumes of Revolution

Exercise A

1 Find the exact volume of the solid generated when each curve is rotated through 360° about the x-axis between the given limits.

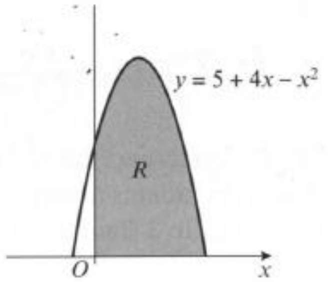
a $y = 10x^2$ between $x = 0$ and $x = 2$

b $y = 5 - x$ between $x = 3$ and $x = 5$

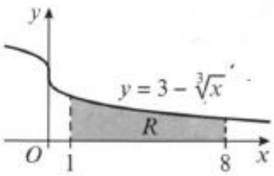
c $y = \sqrt{x}$ between $x = 2$ and $x = 10$

d $y = 1 + \frac{1}{x^2}$ between $x = 1$ and $x = 2$

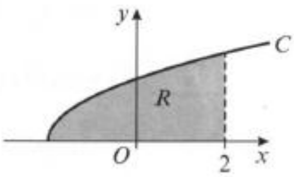
2 The curve shown in the diagram has equation $y = 5 + 4x - x^2$. The finite region R is bounded by the curve, the x-axis and the y-axis. The region is rotated through 2π radians about the x-axis to generate a solid of revolution. Find the exact volume of the solid generated. **(5 marks)**



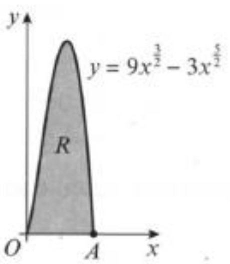
3 The diagram shows the region R which is bounded by the x-axis, the lines $x = 1$ and $x = 8$, and the curve with equation $y = 3 - \sqrt[3]{x}$. The region is rotated through 2π radians about the x-axis. Find the exact volume of the solid generated. **(5 marks)**



4 The diagram shows the curve C with equation $y = \sqrt{x + 2}$. The region R is bounded by the x-axis, the line $x = 2$ and C. The region is rotated through 360° about the x-axis. Find the exact volume of the solid generated. **(5 marks)**



5 The diagram shows a sketch of the curve with equation $y = 9x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$. The region R is bounded by the curve and the x-axis.

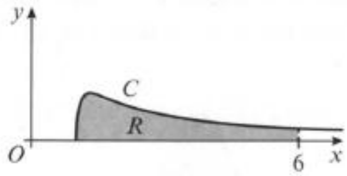


a Find the coordinates of A. **(2 marks)**

The region is rotated through 2π radians about the x-axis.

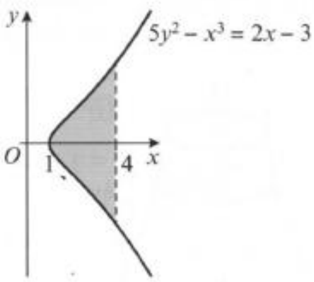
b Find the volume of the solid of revolution generated. **(5 marks)**

6 The curve with equation $y = \frac{\sqrt{3x^4 - 3}}{x^3}$ is shown in the diagram.



The region bounded by the curve C, the x-axis and the line $x = 6$ is shown shaded in the diagram. The region is rotated through 2π radians about the x-axis. Find the volume of the solid generated, giving your answer correct to 3 significant figures. **(6 marks)**

7 The diagram shows the curve with equation $5y^2 - x^3 = 2x - 3$. The shaded region is bounded by the curve and the line $x = 4$. The region is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid generated.

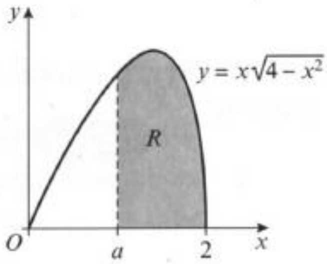


Problem-solving

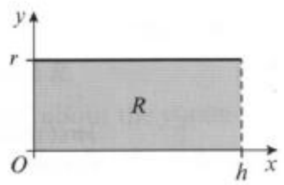
Rearrange the equation to make y^2 the subject.

8 The curve shown in the diagram has equation $y = x\sqrt{4 - x^2}$. The finite region R is bounded by the curve, the x-axis and the line $x = a$, where $0 < a < 2$. The region is rotated through 2π radians about the x-axis to generate a solid of revolution with volume $\frac{657\pi}{160}$

Find the value of a. **(5 marks)**



9 The diagram shows a shaded rectangular region R of length h and width r. The region R is rotated through 360° about the x-axis. Use integration to show that the volume, V, of the cylinder formed is $V = \pi r^2 h$.



Exercise B

1 Find the exact volume of the solid generated when each curve is rotated through 360° about the y-axis between the given limits.

Hint In part **d**, rearrange the expression to make x^2 the subject.

a $x = \frac{1}{2}y + 1$ between $y = 2$ and $y = 5$

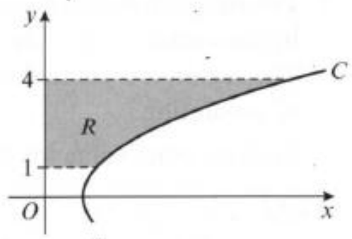
b $y = 2\sqrt{x}$ between $y = 0$ and $y = 1$

c $y = \frac{1}{x}$ between $y = 1$ and $y = 3$

d $y = 2x^2 - 4$ between $x = 5$ and $x = 11$

2 The curve C with equation $x = \frac{1}{2}y^2 + 1$ is shown in the diagram. The region R is bounded by the lines $y = 1$, $y = 4$, the y-axis and the curve C, as shown in the diagram. The region is rotated through 2π radians about the y-axis.

Find the volume of the solid generated. **(6 marks)**

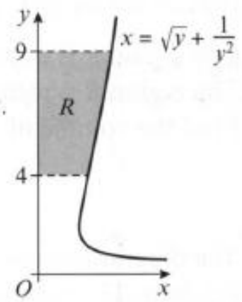


3 The diagram shows the finite region R, which is bounded by the curve $x = \sqrt{y} + \frac{1}{y^2}$, the lines $y = 4$, $y = 9$ and the y-axis.

a Find the exact area of the shaded region. **(3 marks)**

The region R is rotated through 2π radians about the y-axis.

b Use integration to find the volume of the solid generated. Round your answer to 2 decimal places. **(5 marks)**

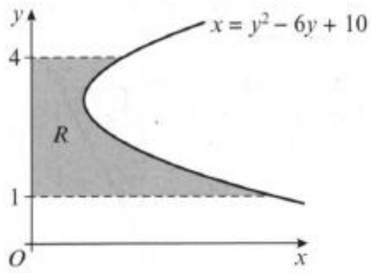


4 The diagram shows the finite region R, which is bounded by the curve $x = y^2 - 6y + 10$, the lines $y = 1$, $y = 4$ and the y-axis.

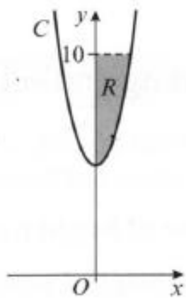
a Find the area of the shaded region R. **(3 marks)**

The region R is rotated through 360° about the y-axis.

b Use integration to find an exact value for the volume of the solid generated. **(5 marks)**



5 The curve C with equation $y = 2x^2 + 5$ is shown in the diagram. The region bounded by the y-axis, the curve C and the line $y = 10$ is shown and shaded in the diagram. The region is rotated 360° about the y-axis. Find the exact volume of the solid generated. **(6 marks)**

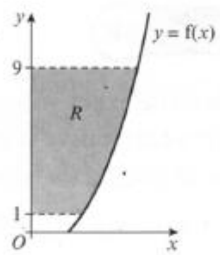


6 $f(x) = x^2 - 2x + 1, x \geqslant 1$

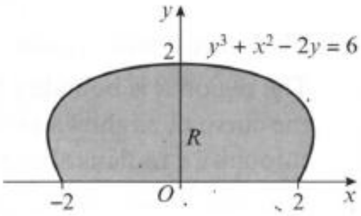
The diagram shows the finite region R bounded by the curve $y = f(x)$, the y-axis and the lines $y = 1$ and $y = 9$.

a Show that the equation $y = f(x)$ can be written as $x^2 = y + 2\sqrt{y} + 1$. **(2 marks)**

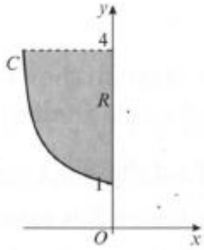
b The region R is rotated through 2π radians about the y-axis. Find the exact volume of the solid generated. **(5 marks)**



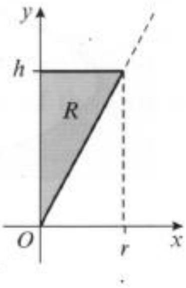
7 The diagram shows the finite region R, which is bounded by the curve $y^3 + x^2 - 2y = 4$ and the x-axis.
The region R is rotated about the y-axis to generate a solid of revolution.
Find an exact value for the volume of the solid. **(5 marks)**



8 Part of the curve C with equation $y^2 = \frac{1}{2x+1}$ is shown in the diagram.
The region R is bounded by the curve, the y-axis and the line $y = 4$.
The region R is rotated 2π radians about the y-axis.
Find the volume of the solid generated. **(5 marks)**



9 The diagram shows a shaded region R in the shape of a right-angled triangle of width r and height h. The region R is rotated through 2π radians about the y-axis. Use integration to show that the volume, V, of the cone formed is given by $V = \frac{1}{3}\pi r^2 h$.



Problem-solving

Start by finding an equation for the line that forms the hypotenuse of the triangle.

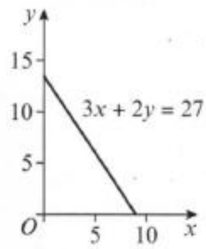
Exercise C

1 The diagram shows the line with equation $3x + 2y = 27$.

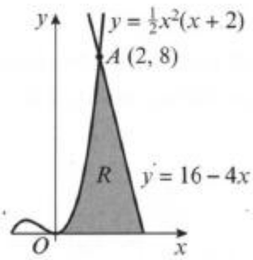
a Use integration to find the volume of revolution when the region is rotated through 360° about the x-axis.

b Use integration to find the volume of revolution when the region is rotated through 360° about the y-axis.

c Use the formula for the cone to check your answers to parts **a** and **b**. Clearly state the radius and the height in each case.



2 The region R is bounded by the curve with equation $y = \frac{1}{2}x^2(x+2)$, the line $y = 16 - 4x$, and the x-axis.

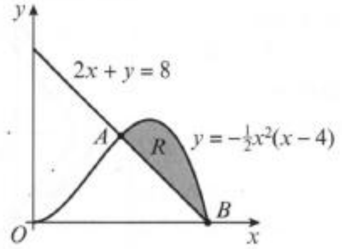


a Show that the coordinates of A are (2, 8). **(1 mark)**

A solid is created by rotating the region through 360° about the x-axis.

b Find the volume of this solid. **(6 marks)**

3 The region R is bounded by the curve with equation $y = -\frac{1}{2}x^2(x - 4)$ and the line $2x + y = 8$.



Problem-solving

You will need to find a volume of revolution then **subtract** the volume of the cone.

a Show that the coordinates of A are (2, 4) and write down the coordinates of B. **(1 mark)**

A solid is created by rotating the region through 360° about the x-axis.

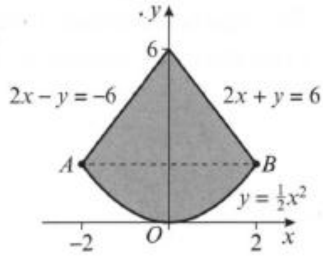
b Find the volume of this solid. **(6 marks)**

4 The shape shown is bounded by the curve $y = \frac{1}{2}x^2$, and the lines $2x + y = 6$ and $2x - y = -6$.

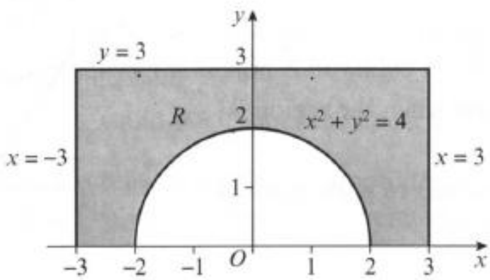
a Find the coordinates of the points A and B. **(2 marks)**

b The shape is rotated about the y-axis to generate a solid of revolution.

Find the volume of the solid generated. **(6 marks)**



5 The region R is bounded by the lines $x = -3$, $x = 3$ and $y = 3$, the curve C with equation $x^2 + y^2 = 4$ and the x-axis. The region is rotated about the y-axis to generate a solid of revolution. Find the volume of the solid generated.

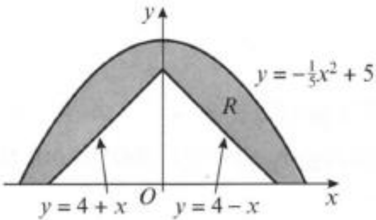


Problem-solving

Find the volume generated by rotating the curve $x^2 + y^2 = 4$ about the x-axis and subtract this from the volume of a suitable cylinder.

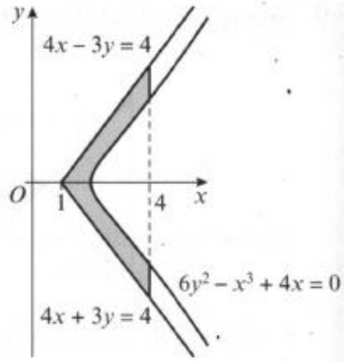
(6 marks)

6 The shaded region R is bounded by the curve $y = -\frac{1}{5}x^2 + 5$, the x-axis and the lines with equations $y = 4 - x$ and $y = 4 + x$.



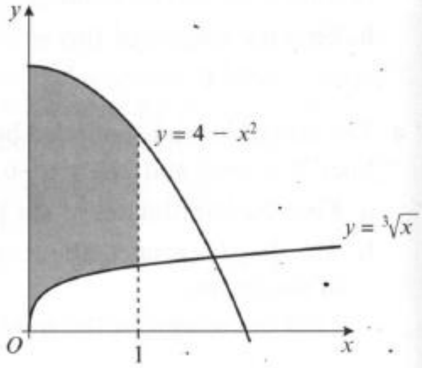
Find the volume of the solid of revolution generated when this region is rotated about the y-axis. **(8 marks)**

7 The shaded region is bounded by the curve C with equation $6y^2 - x^3 + 4x = 0$, $x > 0$, the straight lines $4x - 3y = 4$, $4x + 3y = 4$, and the line $x = 4$. The region is rotated about the x-axis to generate a solid of revolution. Find the exact volume of the solid generated.



8 The shaded region is bounded by the curve with equation $y = 4 - x^2$, the curve with equation $y = \sqrt[3]{x}$, the y-axis and the line with equation $x = 1$.

The region is rotated through 360° about the x-axis. Find the exact volume of the solid generated. **(7 marks)**

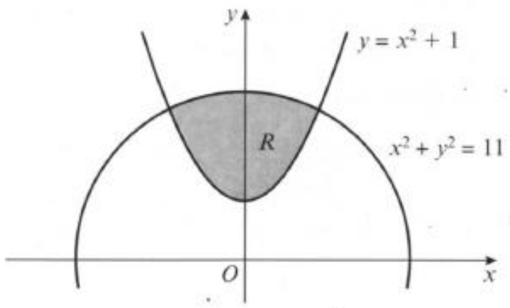


9 The diagram shows the region R bounded by the curve with equation $y = x^2 + 1$ and the curve with equation $x^2 + y^2 = 11$.

a Find the x-coordinates of the points of intersection of the two curves. **(3 marks)**

The region R is rotated through 360° about the x-axis.

b Find the volume of the solid generated, giving your answer correct to 2 decimal places. **(7 marks)**



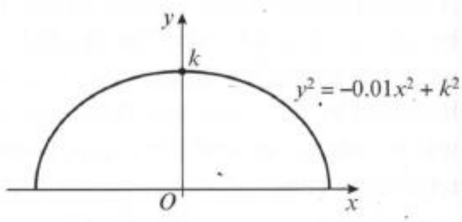
Exercise D

1 The diagram shows the shape of a large tent at a fair. The outside of the tent can be modelled by the equation $y^2 = -0.01x^2 + k^2$. Each unit on the coordinate axes represents 1 metre.

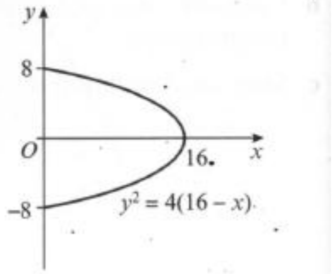
a Suggest a suitable value for k. **(1 mark)**

b Use your value of k to estimate the capacity of the tent. **(5 marks)**

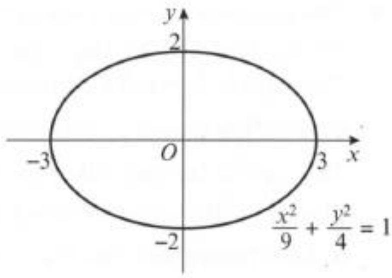
c State one limitation of this model. **(1 mark)**



2 The diagram shows half of the outline of a rugby ball. The outline is modelled by the curve $y^2 = 4(16 - x)$.
The measurements shown are given in centimetres. By rotating the curve through 360° around the y-axis, find the total volume of the rugby ball. **(5 marks)**



3 The cross-section of an egg can be modelled as an ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$, where the dimensions shown are in centimetres.



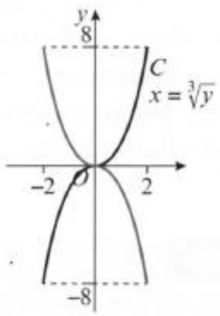
a Calculate the volume of the solid formed by rotating this curve through 360° about the x-axis.

b Show that the solid formed by rotating the curve through 360° about the y-axis has the same volume.

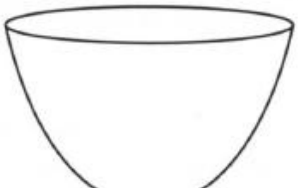
c Say which of these two solids most resembles an egg.

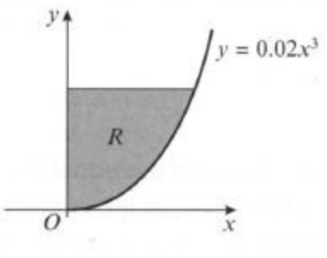
4 The diagram shows the cross-section of an egg timer, which has a height of 16 cm. The shape of the egg timer is modelled as a solid of revolution of a curve C about the y-axis. The curve C has equation $x = \sqrt[3]{y}$.

Sand flows through the egg timer at a rate of 8 cm³/min. The designer wants the egg timer to empty in 5 minutes. Calculate, to 2 decimal places, the height of sand that should be placed in the top half of the egg timer. **(5 marks)**



5 The diagram shows the bowl of an electric stand mixer. The height of the bowl is 18 cm. The shape of the bowl is modelled by rotating the curve with equation $y = 0.02x^3$ through 2π radians about the y-axis.





a Find the diameter of the bowl. **(2 marks)**

b Find the maximum volume of liquid that can be contained within the mixing bowl. **(4 marks)**

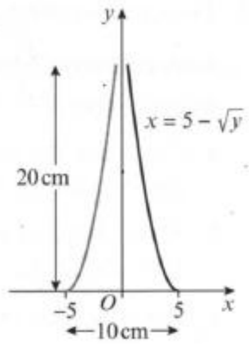
The mixing bowl has a paddle of height 12 cm. The paddle just touches the side of the bowl. In its starting position, the paddle forms a region R, as shown in the diagram.

c Calculate the area of the paddle. **(3 marks)**

The paddle rotates about the y-axis when the mixer is in operation.

d Find the proportion of the total volume contained within the bowl that can be mixed by the paddle. **(4 marks)**

6 The diagram shows a vase with a base width of 10 cm and a height of 20 cm. The edge of the vase is modelled by the equation $x = 5 - \sqrt{y}$. The vase is formed by rotating the shape through 360° about the y-axis.

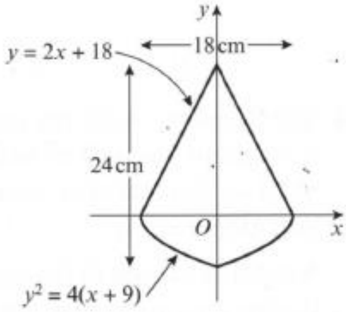


a Use this model to estimate the capacity of the vase. **(5 marks)**

The vase is initially filled to a height of 10 cm. When the flowers are placed in the vase, $50\,cm^3$ of water is displaced.

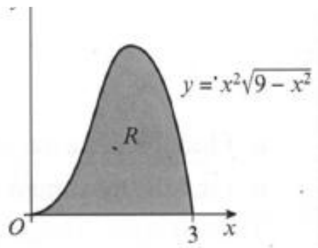
b Determine whether the vase will overflow. **(3 marks)**

7 A circular spinning top is made of solid wood with a width of 18 cm and a height of 24 cm. A cross-section of the spinning top is shown in the diagram. The cross-section is formed by part of the curve with equation $y^2 = 4(x + 9)$ and the straight line with equation $y = 2x + 18$, and is symmetrical about the y-axis. The cross-section is rotated about the y-axis. Find the total volume of wood in the spinning top. **(7 marks)**

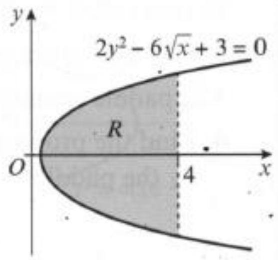


Exercise E

1 The curve shown in the diagram has equation $y = x^2\sqrt{9 - x^2}$. The finite region R is bounded by the curve and the x-axis. The region is rotated through 2π radians about the x-axis to generate a solid of revolution. Find the exact value of the volume of the solid that is generated. **(5 marks)**



2 The diagram shows the curve with equation $2y^2 - 6\sqrt{x} + 3 = 0$. The shaded region is bounded by the curve and the line $x = 4$.



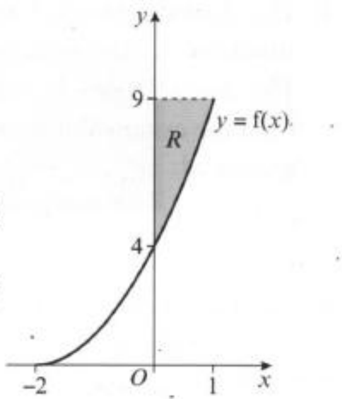
a Find the value of x at the point where the curve cuts the x-axis.

The region is rotated about the x-axis to generate a solid of revolution.

b Find the volume of the solid generated.

3 $f(x) = x^2 + 4x + 4, x \geqslant -2$

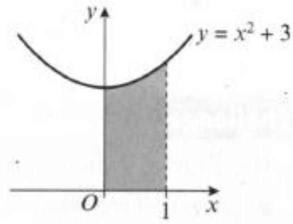
The diagram shows the finite region R bounded by the curve $y = f(x)$, the y-axis and the lines $y = 4$ and $y = 9$.



a Show that the equation $y = f(x)$ can be written as $x^2 = 4 - 4\sqrt{y} + y$. **(2 marks)**

b The region R is rotated through 2π radians about the y-axis. Find the exact volume of the solid generated. **(5 marks)**

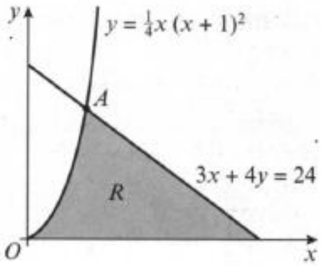
4 The diagram shows the shaded region bounded by the curve with equation $y = x^2 + 3$, the line $x = 1$, the x-axis and the y-axis. Find the volume generated when the region is rotated through 2π radians:



a about the x-axis **(3 marks)**

b about the y-axis. **(4 marks)**

5 The diagram shows the curve with equation $y = \frac{1}{4}x(x + 1)^2$ and the line with equation $3x + 4y = 24$. The line and the curve intersect at the point A.

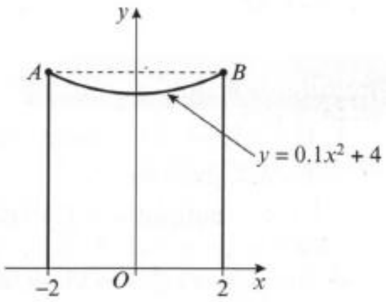


a Show that the coordinates of the point A are (2, 4.5). **(2 marks)**

The shaded region R is bounded by the curve, the line and the x-axis. The region is rotated through 2π radians about the x-axis.

b Find the exact volume of solid generated. **(6 marks)**

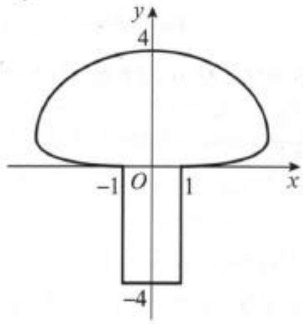
6 The diagram shows a cross-section of a circular golf ball trophy holder. The dimensions shown on the diagram are in centimetres. The cross-section of the trophy is formed by the lines $x = -2$, $x = 2$, the x-axis and the curve with equation $y = 0.1x^2 + 4$. The cross-section is rotated 360° about the y-axis. The trophy holder is to be cast out of solid bronze.



a Use this model to find the volume of bronze needed to make the trophy. **(5 marks)**

b Give one limitation of this model. **(1 mark)**

7 The diagram shows the outline of a circular mushroom. The dimensions on the diagram are in centimetres. The cap of the mushroom is modelled by the curve with equation $\frac{1}{4}x^2 - 8\sqrt{y} + 4y = 0$. The mushroom is formed by rotating the shape shown about the y-axis. Find the exact volume of the mushroom. **(6 marks)**



8 The shaded region is bounded by the curves with equations $y = 2x^2$ and $3y^2 + x^2 - 11y = 0$.

The shaded region is rotated 360° about the y-axis. Find the exact volume of the solid of revolution generated. **(9 marks)**