Answers - Revision – Integration

1.

Question Number	Scheme	Marks
(a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1-2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	$[\ R^2 = 4^2 + 6^2,\ R\sin\alpha = 4,\ R\cos\alpha = 6\ ]$	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α; $\alpha = 0.588$ (allow 33.7°)	M1 A1 (4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \dfrac{3}{\sqrt{52}} = 0.4160..$ (24.6°)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6°),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1 (5)
		[15]

2.

Question Number	Scheme	Marks
	(a) $R=25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = (\text{awrt})73.7^0$	M1A1
		(3)
	(b) $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$	M1
	$2x + their\,'\alpha' = 60^0$	A1
	$2x + their\,'\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$	M1
	$x = awrt\ 113.1^0, 173.1^0$	A1A1
		(5)
	(c)	
	Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x \cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$	
	$= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
	(d)	
	$14\cos^2 x - 48\sin x\cos x = R\cos(2x+\alpha) + 7$	
	Maximum value ='R'+'c'	M1
	= 32 cao	A1
		(2)
		(12 marks)

3.

Question Number	Scheme	Marks
(a)	$\text{cosec}2x+\cot 2x=\frac{1}{\sin 2x}+\frac{\cos 2x}{\sin 2x}$	M1
	$=\frac{1+\cos 2x}{\sin 2x}$	M1
	$=\frac{1+2\cos^2 x-1}{2\sin x\cos x}$	
	$=\frac{2\cos^2 x}{2\sin x\cos x}$	M1 A1
	$=\frac{\cos x}{\sin x}=\cot x$	A1*
		(5)
(b)	$\text{cosec}(4\theta+10°)+\cot(4\theta+10°)=\sqrt{3}$	
	$\cot(2\theta\pm..°)=\sqrt{3}$	M1
	$2\theta\pm...=30°\Rightarrow\theta=12.5°$	dM1, A1
	$2\theta\pm...=180+PV°\Rightarrow\theta=..°$	dM1
	$\theta=102.5°$	A1
		(5)

4.

Question Number	Scheme		Marks
(i) (a)	$2\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}=\frac{5}{\sin x}$		B1
	Uses common denominator to give $2\sin^2 x-\cos^2 x=5\cos x$		M1
	Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x)-\cos^2 x=5\cos x$		M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, b = 5, c = -2)		A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give $\cos x=$		M1
	$\cos x=\frac{1}{3}$ only (rejects $\cos x$ = -2)		A1
	So x = 1.23 or 5.05		dM1A1 (4)
(ii)	Either	Or	
	$\tan\theta+\cot\theta\equiv\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$	$\tan\theta+\cot\theta\equiv\tan\theta+\frac{1}{\tan\theta}$	B1
	$\equiv\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	$\equiv\frac{\tan^2\theta+1}{\tan\theta}$	M1
	$\equiv\frac{2}{\sin 2\theta}$	$\equiv\frac{1}{\cos^2\theta\times\frac{\sin\theta}{\cos\theta}}\equiv\frac{2}{\sin 2\theta}$	M1
	$\equiv 2\text{cosec}2\theta$ (so $\lambda=2$)	$\equiv 2\text{cosec}2\theta$ (so $\lambda=2$)	A1 (4)
			12 marks

5.

(*a*)	Method for finding $\sin A$	M1	
	$\sin A = -\frac{\sqrt{7}}{4}$	A1 A1	
	Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)		
	Use of $\sin 2A \equiv 2\sin A\cos A$	M1	
	$\sin 2A = -\frac{3\sqrt{7}}{8}$ or equivalent exact	A1√	(5)
	Note: ± f.t. Requires exact value, dependent on 2nd M		
(*b*)(i)	$\cos\left(2x+\frac{\pi}{3}\right)+\cos\left(2x-\frac{\pi}{3}\right) \equiv \cos 2x\cos\frac{\pi}{3} - \sin 2x\sin\frac{\pi}{3} +$ $\cos 2x\cos\frac{\pi}{3} + \sin 2x\sin\frac{\pi}{3}$	M1	
	$\equiv 2\cos 2x\cos\frac{\pi}{3}$	A1	
	[This can be just written down (using factor formulae) for M1A1]		
	$\equiv \cos 2x$ AG	A1*	(3)
	Note: M1A1 earned, if $\equiv 2\cos 2x\cos\frac{\pi}{3}$ just written down, using factor theorem Final A1* requires some working after first result.		
(*b*)(ii)	$\frac{dy}{dx} = 6\sin x\cos x - 2\sin 2x$	B1 B1	
	or $6\sin x\cos x - 2\sin\left(2x+\frac{\pi}{3}\right) - 2\sin\left(2x-\frac{\pi}{3}\right)$		
	$= 3\sin 2x - 2\sin 2x$	M1	
	$= \sin 2x$ AG	A1*	(4)
	Note: First B1 for $6\sin x\cos x$; second B1 for remaining term(s)		
			(12 marks)

6.

Question Number	Scheme		Marks	
(a)(i)	$\sin 3\theta = \sin(2\theta+\theta)$			
	$= \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1-2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ ✱	cso	A1	(4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	(5)
(b)	$\sin 15° = \sin(60°-45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$		M1	
	$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱	cso	A1	(4)
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45°-30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$		M1	
	$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱	cso	A1	(4)
	② Using $\cos 2\theta = 1-2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6}-\sqrt{2})\right)^2 = \frac{1}{16}(6+2-2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱	cso	A1	(4)

7.

Question Number	Scheme	Marks
(a)	$\frac{1}{\sin2\theta}-\frac{\cos2\theta}{\sin2\theta}=\frac{1-\cos2\theta}{\sin2\theta}$	M1
	$=\frac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ cso	A1* (4)
(b)(i)	$\tan15° =\frac{1}{\sin30°}-\frac{\cos30°}{\sin30°}$	M1
	$\tan15° =\frac{1}{\frac{1}{2}}-\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=2-\sqrt{3}$ cso	dM1 A1* (3)
(b)(ii)	$\tan2x=1$	M1
	$2x=45°$	A1
	$2x=45°+180°$	M1
	$x=22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
	Alt for (b)(i) $\tan 15° = \tan(60°-45°)$ *or* $\tan(45°-30°)$	12 Marks
	$\tan 15° = \frac{\tan60-\tan45}{1+\tan60\tan45}$ *or* $\frac{\tan45-\tan30}{1+\tan45\tan30}$	M1
	$\tan 15° = \frac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan15° = 2-\sqrt{3}$	A1*

8.

Question No	Scheme	Marks
	(a) $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1
	$=\frac{tanA+tanB}{1-tanAtanB}$	A1 *
		(4)
	(b) $\tan\left(\theta+\frac{\pi}{6}\right)=\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$	M1
	$=\frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$	M1
	$=\frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$	A1 *
		(3)
	(c) $\tan\left(\theta+\frac{\pi}{6}\right)=\tan(\pi-\theta).$	M1
	$\left(\theta+\frac{\pi}{6}\right)=(\pi-\theta)$	dM1
	$\theta=\frac{5}{12}\pi$	ddM1 A1
	$\tan\left(\theta+\frac{\pi}{6}\right)=\tan(2\pi-\theta)$	dddM1
	$\theta=\frac{11}{12}\pi$	A1
		(6)
		(13 MARKS)

9.

Question Number	Scheme	Marks
(i)	$\frac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5\Rightarrow\tan(2x+32°)=5$	B1
	$\Rightarrow x=\frac{\arctan 5-32°}{2}$	M1
	$\Rightarrow x=$ awrt $23.35°,-66.65°$	A1A1
		(4)
(ii)(a)	$\tan(3\theta-45°)=\frac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\frac{\tan 3\theta-1}{1+\tan 3\theta}$	M1A1*
		(2)
(b)	$(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$	
	$\Rightarrow\tan(\theta+28°)=\tan(3\theta-45°)$	B1
	$\theta+28°=3\theta-45°\Rightarrow\theta=36.5°$	M1A1
	$\theta+28°+180°=3\theta-45°\Rightarrow\theta=126.5°$	dM1A1
		(5)

10.

Question Number	Scheme	Marks
(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = \text{'}-77.1...\text{'} \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

11.

Question Number	Scheme	Marks
(a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	M1A1
		(3)
(b) (i)	Max Value = 2.5	B1 $\sqrt{}$
(ii)	$\sin(\theta - 0.6435) = 1$ or $\theta - \text{their } \alpha = \frac{\pi}{2}$; $\Rightarrow \theta = \text{awrt } 2.21$	M1;A1 $\sqrt{}$
		(3)
(c)	$H_{Max} = 8.5$ (m)	B1 $\sqrt{}$
	$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$ or $\frac{4\pi t}{25} =$ their (b) answer ; $\Rightarrow t = \text{awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = \text{awrt } 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517...\text{ or } 2.730076...^c\right\}$	ddM1
	Times $= \{14:06, 18:43\}$	A1 (6)
		[15]

12.

Question Number	Scheme	Marks
(a)	$R=\sqrt{(6^2+2.5^2)}=6.5$	B1
	$\tan\alpha=\frac{2.5}{6}, \Rightarrow \alpha=\text{awrt }0.395$	M1A1
		(3)
(b)	$(0,6)$,	B1
	awrt $(1.97,0)$ $(5.11,0)$	M1A1
		(3)
(c)	$H_{\max}=18.5, H_{\min}=5.5$	M1A1A1
		(3)
(d)	Sub $H=16$ and proceed to $'6.5'\cos\left(\frac{2\pi t}{52}\pm'0.395'\right)=4$	M1
	$\left(\frac{2\pi t}{52}-'0.395'\right)=\text{awrt }0.91$	A1
	$t=(awrt\,0.908\pm'0.395')\times\frac{52}{2\pi}=11\ (10.78)$	dM1A1
	$\left(\frac{2\pi t}{52}\pm'0.395'\right)=awrt\,2\pi-0.908\Rightarrow t=48\ (47.75)$	ddM1A1
		(6)
		(15 marks)

13.

Question Number	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=2\Rightarrow\alpha=\text{awrt }1.107$	M1A1
		(3)
(b)(i)	$'40+9R^2'=85$	M1A1
(ii)	$\theta=\frac{\pi}{2}+1.107\Rightarrow\theta=\text{awrt }2.68$	B1ft
		(3)
(c)(i)	6	B1
(ii)	$2\theta-'1.107'=3\pi\Rightarrow\theta=\text{awrt }5.27$	M1A1
		(3)
		(9 marks)