

Revision - Trigonometry

1.

(a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A. \quad (2)$$

(b) Show that

$$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta(4 \cos \theta + 6 \sin \theta - 3). \quad (4)$$

(c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (4)(d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate. (5)

2.

$$f(x) = 7 \cos 2x - 24 \sin 2x$$

Given that $f(x) = R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,(a) find the value of R and the value of α . (3)

(b) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180^\circ$, giving your answers to 1 decimal place. (5)(c) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a , b , and c are constants to be found. (2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x \quad (2)$$

3.

(a) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n^\circ, \quad n \in \mathbb{Z} \quad (5)$$

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180^\circ$,

$$\operatorname{cosec} (4\theta + 10^\circ) + \cot (4\theta + 10^\circ) = \sqrt{3}$$

You must show your working.

(Solutions based entirely on graphical or numerical methods are not acceptable.) (5)

4.

(i) (a) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a , b and c .

(4)

(b) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \quad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ .

(4)

5.

(a) Given that $\cos A = \frac{3}{4}$, where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$. (5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$. (3)

Given that

$$y = 3 \sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right).$$

(ii) show that $\frac{dy}{dx} = \sin 2x$. (4)

6.

(a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \quad (4)$$

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π .

(5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}). \quad (4)$$

7.

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \quad \theta \neq 90n^\circ, \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence, or otherwise,

(i) show that $\tan 15^\circ = 2 - \sqrt{3}$, (3)

(ii) solve, for $0 < x < 360^\circ$,

$$\operatorname{cosec} 4x - \cot 4x = 1 \quad (5)$$

8.

(a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad (4)$$

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta} \quad (3)$$

(c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π .

(6)

9.

(i) Using the identity for $\tan(A \pm B)$, solve, for $-90^\circ < x < 90^\circ$,

$$\frac{\tan 2x + \tan 32^\circ}{1 - \tan 2x \tan 32^\circ} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (a) Using the identity for $\tan(A \pm B)$, show that

$$\tan(3\theta - 45^\circ) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \quad \theta \neq (60n + 45)^\circ, n \in \mathbb{Z} \quad (2)$$

(b) Hence solve, for $0 < \theta < 180^\circ$,

$$(1 + \tan 3\theta) \tan(\theta + 28^\circ) = \tan 3\theta - 1 \quad (5)$$

10.

$$g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta$$

Given that $g(\theta) = R \cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the exact value of R and the value of α to 2 decimal places. (3)

(b) Hence solve, for $-90^\circ < \theta < 90^\circ$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1$$

giving your answers to one decimal place. (5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k . (2)

11.

(a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. (3)

(b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs. (3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs. (3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. (6)

12.

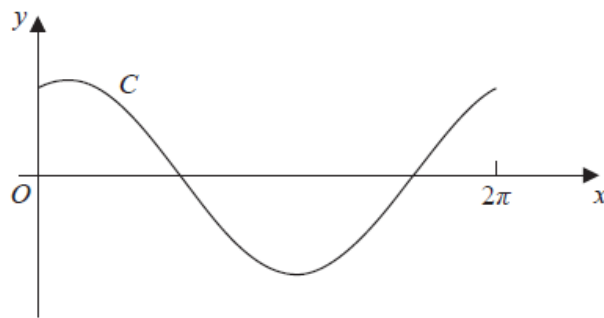


Figure 1

Figure 1 shows the curve C , with equation $y = 6 \cos x + 2.5 \sin x$ for $0 \leq x \leq 2\pi$

(a) Express $6 \cos x + 2.5 \sin x$ in the form $R \cos(x - a)$, where R and a are constants

with $R > 0$ and $0 < a < \frac{\pi}{2}$. Give your value of a to 3 decimal places. (3)

(b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes. (3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6 \cos\left(\frac{2\pi t}{52}\right) + 2.5 \sin\left(\frac{2\pi t}{52}\right), \quad 0 \leq t \leq 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(c) the maximum and minimum values of H predicted by the model, (3)

(d) the values for t when $H = 16$, giving your answers to the nearest whole number.

[You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.]

(6)

13.

- (a) Express $\sin \theta - 2 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α , in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3 \sin \theta - 6 \cos \theta)^2$$

(b) Find

- (i) the maximum value of $M(\theta)$,
- (ii) the smallest value of θ , in the range $0 < \theta \leq 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2 \cos 2\theta)^2}$$

(c) Find

- (i) the maximum value of $N(\theta)$,
- (ii) the largest value of θ , in the range $0 < \theta \leq 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(Solutions based entirely on graphical or numerical methods are not acceptable.)
