Revision - Integration

1.

(i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx = \lambda\int_0^{\frac{\pi}{3}} \sin^2\theta\,d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

2.

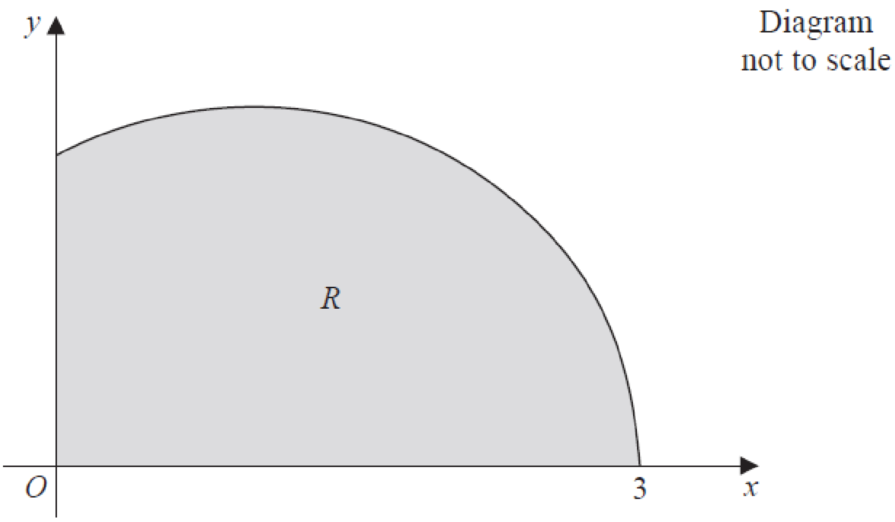

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,dx = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

3.

Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

(6)

4.

(i) Find

$$\int x e^{4x}\,dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

5.

Use integration to find the exact value of $\int_{\frac{1}{16}\pi}^{\frac{1}{8}\pi} (9 - 6\cos^2 4x)\,dx$. **[5]**

6.

Use the substitution $u = x^2 - 2$ to find $\int \frac{6x^3 + 4x}{\sqrt{x^2 - 2}} \, dx$. **[6]**

7.

(i) Express $\frac{16 + 5x - 2x^2}{(x+1)^2(x+4)}$ in partial fractions. **[5]**

(ii) It is given that

$$\frac{dy}{dx} = \frac{(16 + 5x - 2x^2)y}{(x+1)^2(x+4)}$$

and that $y = \frac{1}{256}$ when $x = 0$. Find the exact value of y when $x = 2$. Give your answer in the form Ae^n. **[7]**

8.

(i) Use the quotient rule to show that the derivative of $\frac{\cos x}{\sin x}$ is $\frac{-1}{\sin^2 x}$. **[2]**

(ii) Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{\sqrt{1 + \cos 2x}}{\sin x \sin 2x} dx = \frac{1}{2}(\sqrt{6} - \sqrt{2})$. **[6]**

9.

By first using the substitution $t = \sqrt{x+1}$, find $\int e^{2\sqrt{x+1}} dx$. **[6]**

10.

(i) Use division to show that $\frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}$. **[3]**

(ii) Find $\int_1^2 6t^2 \ln(t+2) \, dt$. Give your answer in the form $A + B\ln 3 + C\ln 4$. **[6]**

11.

Express $\frac{2 + x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\int_0^{\frac{1}{4}} \frac{2 + x^2}{(1+2x)(1-x)^2} dx = \frac{1}{2}\ln\frac{3}{2} + \frac{1}{3}$. **[9]**

12.

Find $\int x^8 \ln(3x)\,dx$. **[5]**

13.

(i) Show that $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x}\right) dx$, giving your answer in the form $a \ln b$. **[5]**

14.

Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \frac{\ln x}{x(1+\ln x)^2}\,dx$. **[6]**

15.

Find $\int x \cos 3x\,dx$. **[4]**

16.

Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5}\,dx$. **[7]**

17.

(i) Given that $y = \ln(1 + \sin x) - \ln(\cos x)$, show that $\dfrac{dy}{dx} = \dfrac{1}{\cos x}$. **[4]**

(ii) Using this result, evaluate $\displaystyle\int_0^{\frac{1}{3}\pi} \sec x\,dx$, giving your answer as a single logarithm. **[3]**

18.

(i) Use algebraic division to express $\dfrac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 - x - 6}$, where A, B, C and D are constants. **[4]**

(ii) Hence find $\displaystyle\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx$, giving your answer in the form $a + \ln b$. **[7]**
