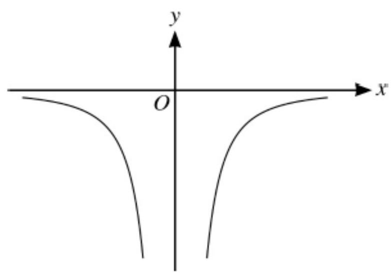
1.

Question	Scheme	Marks
(a)	$(g(-2)) = 4\times-8-12\times4-15\times-2+50$	M1
	$g(-2)=0 \Rightarrow (x+2)$ is a factor	A1
		(2)
(b)	$4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$	M1 A1
	$=(x+2)(2x-5)^2$	M1 A1
		(4)
(c)	(i) $x \leqslant -2,\ x=2.5$	M1 A1ft
	(ii) $x=-1,\ x=1.25$	B1ft
		(3)

2.

Question	Answer	Marks	AO	Guidance
(a)		B1 [1]	1.1	Curve in both quadrants: • Correct shape, symmetrical, not touching axis • Asymptote the axes • Not finite • Allow slight movement away from asymptote at one end but not more
(b)	$y=-\frac{1}{(x-2)^2}$	M1 A1 [2]	1.1 2.5	$(y=)-\frac{1}{(x-2)^2}$ or $(y=)-\frac{1}{(x+2)^2}$ Fully correct, must include '$y=$'
(c)	$\left(\frac{1}{2},-2\right)$	B2 [2]	1.1 1.1	B1 for each coordinate

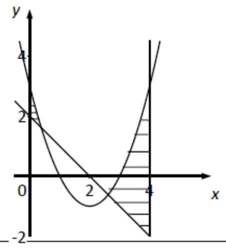
3.

Question			Answer	Marks	AOs	Guidance
	(i)		$\frac{2}{3+x-4}$ or $\frac{2}{3+x+4}$	M1	1.1	Translates curve by $+/-$ 4 parallel to the x-axis
			$y=\frac{2}{x-1}$	A1	1.1	Fully correct, must have $y=$
				[2]		
	(ii)		Stretch	B1	1.2	Must use stretch/stretched/stretching…
			Scale factor $\frac{5}{2}$ parallel to the y-axis	B1	2.5	Allow "factor" or "SF" for "scale factor". Allow "vertically", "in the y direction". Do not accept "in/on/across/up/along the y axis", "in the positive y direction", "SF 5/2 units"
				[2]		

4.

Question	Scheme	Marks
(a)	Attempts $f(4)=2\times4^3-13\times4^2+8\times4+48$	M1
	$f(4)=0\Rightarrow(x-4)$ is a factor	A1
		(2)
(b)	$2x^3-13x^2+8x+48=(x-4)(2x^2...x-12)$	M1
	$=(x-4)(2x^2-5x-12)$	A1
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1
	$f(x)=(x-4)^2(2x+3)\Rightarrow f(x)=0$ has only two roots, 4 and -1.5	A1
		(4)
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1
		(2)
(d)	For sight of $k=\pm4,\pm\frac{3}{2}$	M1
	$k=4,-\frac{3}{2}$	A1ft
		(2)

5.

(a)	$x^2 - 3x + 1 \Rightarrow x^2 - 4x + 3 = -x + 2$	M1	1.1	Attempt form equation of form $x^2 - 4x + 3 = mx + c$ NB $x^2 - 3x + 1 = x^2 - 4x + 3$: M0 unless this leads to $y = mx + c$ seen
	$m = -1,\ c = 2$ or $y = -x + 2$	A1 [2]	1.1	
(b)	Line $y = -x + 2$ drawn $x = 0.4\ (\pm 0.1),\ \ x = 2.6\ (\pm 0.1)$	M1 A1 [2]	1.1 2.2a	Good attempt at draw their line from (a) Ignore y-coords cao NB, correct answers do NOT score marks unless they clearly come from the correct line seen, except:. SC: correct answers from graph of $y = x^2 - 3x + 1$ B0B1
(c)		B1ft B1ft B1 [3]	1.1 1.1 1.1	At least one region indicated that is: wholly above the line $y = -x + 2$, ft their line, no omission Follow only correct line or their line from (a) wholly below the curve $y = x^2 - 4x + 3$, no omissions Follow their line as drawn with its shading All correct cao Accept any correct indication, eg shading in, shading out, arrows, letters etc

6.

$(x-2)^2 + \left(y + \frac{k}{2}\right)^2$ Allow + or – in both	M1*	3.1a
$(x-2)^2 + \left(y + \frac{k}{2}\right)^2 + 12 - 4 - \frac{k^2}{4}$ oe; ignore RHS	A1	1.1
$\frac{k^2}{4} - 8 = 1$ or $-12 + 4 + \frac{k^2}{4} = 1$ oe ft const term depM1*	M1	2.1
$k^2 = 36$ $k = \pm 6$	A1 [4]	1.1

7.

Answer		Mks	AO	Guidance	
DR $y - 1 = -2(x-2)$	or $y = -2x + c$ & sub (2, 1)	M1	3.1a	**If no wking seen, no marks** or $y - 1 = 2(x-(-2))$ or solve $y=-2x+5$ & $y=2x+5$	Alt method using proportion: Centre is on y-axis, not (0, 1) (may be implied) M1 $\frac{c-1}{2} = 2$ or $c = 1 + 2\times 2$
$y = -2x + 5$	$c = 5$	A1	1.1	$y = 2x + 5$ or $c = 5$	$c = 5$ A1
Centre is (0, 5)		A1	3.2a	stated or implied	Centre is (0, 5) A1
$r = \sqrt{2^2 + 4^2}$ $= \sqrt{20}$		M1	1.1a	or $r^2 = 2^2 + 4^2$ or ft their centre $= 20$	
$x^2 + (y-5)^2 = 20$ oe		M1	1.2	or $a = 0,\ b = -10,\ c = 5$ ft their centre and rad^2 ($\neq 0$), however found	
$x^2 + y^2 - 10y + 5 = 0$		A1 [6]	1.1	cao	

8.

Question	Scheme	Marks	AOs
(a)	Uses $15=\frac{1}{2}\times 5\times 10\times \sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times 10\times 5\times "-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	

9.

(a)		$\frac{1}{2}xy\sin 60=\sqrt{3}(x+y)$ Must have xy, not ab May be implied by next line $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)xy=\sqrt{3}(x+y)$ **oe** $\Rightarrow 4x+4y=xy$ **(AG)**	**M1** **A1** **[2]**	**1.1a** **2.1**	Verification method: eg substitute $4x+4y$ for xy in $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)xy=\sqrt{3}(x+y)$ M1 obtain $\sqrt{3}(x+y)=\sqrt{3}(x+y)$ A1
(b)		Angle ABC is 90°	**B1***	**3.1a**	May be implied by correct trig statement
		$\cos 60=\frac{x}{y}$ or $\frac{x}{y}=\frac{1}{2}$ or $y=2x$ $4x+4(2x)=x(2x)$ oe or $4(\frac{y}{2})+4y=y\times\frac{y}{2}$ oe $(\Rightarrow x(x-6)=0)$ or $\Rightarrow y(y-12)=0)$ $x=6,\ y=12$	**B1** **M1** **A1** **[4]**	**1.1** **1.1** **1.1**	or any correct x, y equn eg $\frac{x}{\sin 30}=\frac{y}{\sin 90}$ or $\sqrt{3}(x+2x)=\frac{1}{2}\times x\times 2x\times\frac{\sqrt{3}}{2}$ No need to consider $x=0$ or $y=0$

10.

Question	Scheme	Marks	AOs
(a)	$\frac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\frac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\frac{5}{3}$	M1	2.1
	$x=$ awrt $121°, 301°$	A1 A1	1.1b 1.1b
		(3)	

11.

(a)	$6\left(1-\sin^2\theta\right)=\frac{\sin\theta}{\cos\theta}(\cos\theta)+4$	M1	3.1a	Correct use of both $\cos^2\theta=1-\sin^2\theta$ and $\tan\theta=\frac{\sin\theta}{\cos\theta}$
	$6-6\sin^2\theta=\sin\theta+4 \Rightarrow 6\sin^2\theta+\sin\theta-2=0$	A1	2.1	AG
		[2]		
(b)	$(2\sin\theta-1)(3\sin\theta+2)$	M1	1.1a	Attempt to solve 3-term quadratic
	Critical values occur when $\sin\theta=\frac{1}{2}$ and $\sin\theta=-\frac{2}{3}$	B1	1.1	
	Critical values are $\theta=30,150,222,318$	B1	1.1	Any three correct critical values
	$0<\theta<30$ or $150<\theta<222$ or $318<\theta<360$	B1	1.1	B1 for one correct interval – 3sf or better (condone use of x)
		A1	2.5	Cao (all three intervals) – 3 sf or better For those that have $\sin\theta=-\frac{1}{2}$ and $\sin\theta=\frac{2}{3}$ can score M1 (if **DR** seen) then **SC B1** for one 'correct' interval (condone $\le$ oe) or **SC B2** for all three 'correct' intervals which are $\theta<42, 138<\theta<210, \theta>330$ (so max. 3/5)
		[5]		

12.

Question	Scheme	Marks	AOs
(a)	$4\cos\theta-1=2\sin\theta\tan\theta \Rightarrow 4\cos\theta-1=2\sin\theta\times\frac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta-\cos\theta=2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta-\cos\theta=2\left(1-\cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta-\cos\theta-2=0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x)=\frac{2}{3},-\frac{1}{2}$	B1	1.1b
	$x=\frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x=40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	

13.

(a)		$\log_2 x^2 \left(= \log_2 (kx-1)+3\right)$	B1	1.2	Using $a\log b = \log\left(b^a\right)$
		$\log_2\left(\frac{x^2}{kx-1}\right)=3$	M1*	2.1	Re-arranging and correctly combining both log terms
		$\frac{x^2}{kx-1}=2^3$	Dep*M1	1.1	Correctly remove logs
		$x^2=8(kx-1)$ $x^2-8kx+8=0$	A1	1.1	AG
			[4]		
(b)		$b^2-4ac=0 \Rightarrow (-8k)^2-4(1)(8)=0$	M1	3.1a	Use of $b^2-4ac=0$
		$k=(\pm)\frac{1}{\sqrt{2}}$	A1	1.1	oe exact
		$k=\frac{1}{\sqrt{2}} \Rightarrow x=2\sqrt{2}$	A1	2.2a	BC oe exact
		$k=-\frac{1}{\sqrt{2}} \Rightarrow x=-2\sqrt{2}$ and as $\log_2 x$ is only defined for $x>0$ so $x \neq -2\sqrt{2}$	A1	3.2b	BC oe statement for rejection of negative value of x (allow decimal argument)
			[4]		

14.

Question Number	Scheme	Marks
(a)	$\log_x 64 = 2 \quad \Rightarrow \quad 64 = x^2$	M1
	So $x = 8$	A1 (2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$	M1
	$\log_2\left[\frac{11-6x}{(x-1)^2}\right]=3$	M1
	$\frac{11-6x}{(x-1)^2}=2^3$	M1
	$\{11-6x=8\left(x^2-2x+1\right)\}$ and so $0=8x^2-10x-3$	A1
	$0=(4x+1)(2x-3) \Rightarrow x=\ldots$	dM1
	$x=\frac{3}{2},\left[-\frac{1}{4}\right]$	A1 (6)
		[8]

15.

(a)	(i)	5460 (3 sf)	B1 [1]	1.1	
(a)	(ii)	$9000 = 100e^t$ $t = \ln 90$ $= 4.50$ (3 sf) Allow 4.5 ISW	 M1 A1 [2]	 3.1a 1.1	 May be implied by answer Ignore units. Decimal answer needed
(b)	(i)	$\log_{10} P = \log_{10}(ka^t)$ $\log_{10} P = \log_{10} k + \log_{10}(a^t)$ $\log_{10} P = \log_{10} k + t\log_{10} a$	 M1 A1 [2]	 1.1 1.1	No marks yet At least two terms correct, may be implied by next line All correct, in this form
(b)	(ii)	Points plotted correctly ±0.1 Line of best fit drawn, between (1, 2.0) and (1, 2.4) and between (5, 4.2) and (5, 4.5)	B1 B1f [2]	1.1 1.1	NB. May be implied by correct line of best fit ft reasonable line through their points
(b)	(iii)	Read off c and attempt 10^c. May be implied by value of k $k = 19.9$ to 63.1 Attempt gradient of their graph AND correct ft equation in a. May be implied by value of a $a = 3.16$ to 5.01 (3 sf)	M1 A1 M1 A1 [4]	3.1a 2.1 1.1 1.1	ft their line. Probably $c = 1.3$ to 1.8, $k = 10^{1.3}$ to $10^{1.8}$ ft their line. Probably $m = 0.5$ to 0.7 AND $\log_{10} a = 0.5$ to 0.7 OR $a = 10^{0.5}$ to $10^{0.7}$ scores NB Use of two points and simultaneous equations: no marks unless the two points used are on their line of best fit. If first method used for k or a and then one point substituted in equation to find the other letter, no marks for second letter unless point used is on line of best fit.

16.

(a)		$3x^2 - 12x + 9 = 0$ $x = 3$ or 1 $(3, 0)$ and $(1, 4)$	M1 A1 A1f A1 [4]	1.1a 1.1 1.1 1.1	Attempt differentiate & =0 May be implied Correct equation. May be implied by ans BC Allow When $x = 3$, $y = 0$; when $x = 1$, $y = 4$
(b)		Sketch (drawn in this part) of "+ve" cubic with two SPs, roughly correct shape, or just two TPs shown OR: f(1) = 0 and f(3) = 0, find k for each $k > 0$ or $k < -4$	M1 A1f [2]	3.1a 2.2a	Subst x=1 & x=3 into $y = x^3 - 6x^2 + 9x$ ft (a) ft their (a) Correct ans: M1A1

17.

Question	Scheme	Marks	AOs
(a)	$x^n \rightarrow x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	

18.

Question Number	Scheme		Marks
(a)	$\{V=\}\ 2x^2y=81$	$2x^2y=81$	B1 oe
	$\{L=\ 2(2x+x+2x+x)+4y \Rightarrow L=\ 12x+4y\}$		
	$y=\dfrac{81}{2x^2} \Rightarrow L=12x+4\left(\dfrac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L=12x+\dfrac{162}{x^2}$ **AG**	Correct solution only. **AG.**	A1 cso
			[3]
(b)	$\dfrac{dL}{dx}=12-\dfrac{324}{x^3}\quad\{=12-324x^{-3}\}$	Either $12x \to 12$ or $\dfrac{162}{x^2} \to \dfrac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\dfrac{dL}{dx}=\right\}12-\dfrac{324}{x^3}=0 \Rightarrow x^3=\dfrac{324}{12};=27 \Rightarrow x=3$	$L'=0$ and "their $x^3=\pm$ value" or "their $x^{-3}=\pm$ value"	M1;
		$x=\sqrt[3]{27}$ or $x=3$	A1 cso
	$\{x=3,\}\ L=12(3)+\dfrac{162}{3^2}=54$ (cm)	Substitute candidate's value of $x(\neq 0)$ into a formula for L.	ddM1
		54	A1 cao
			[6]
(c)	$\{$For $x=3\}$, $\dfrac{d^2L}{dx^2}=\dfrac{972}{x^4}>0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\dfrac{972}{x^4}$ and >0 and conclusion.	A1 **[2]**
			11

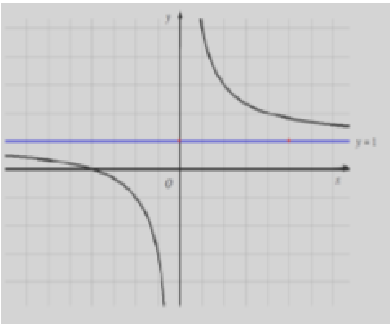
19.

(i)	**(a)**	$\mathbf{c}-\mathbf{a}$ oe	**B1** **[1]**	**1.2**	
(i)	**(b)**	$\mathbf{a}+\frac{1}{2}(\mathbf{c}-\mathbf{a})$ or $\mathbf{c}+\frac{1}{2}(\mathbf{a}-\mathbf{c})$	**M1**	**3.1a**	$\mathbf{a}+\frac{1}{2}$ their (a) or $\mathbf{c}-\frac{1}{2}$ their (a)
		$=\frac{1}{2}(\mathbf{a}+\mathbf{c})$ or $\frac{1}{2}\mathbf{a}+\frac{1}{2}\mathbf{c}$	**A1** **[2]**	**1.1b**	Correct ans without wking: M1A1
(ii)		$\overrightarrow{OB}=(\mathbf{a}+\mathbf{c})$	**M1**	**3.1a**	$\overrightarrow{PB}=\mathbf{a}+\frac{1}{2}(\mathbf{c}-\mathbf{a})$ or $\mathbf{a}+\frac{1}{2}$ their (i)(a) or $\mathbf{c}+\frac{1}{2}(\mathbf{a}-\mathbf{c})$ ($=\frac{1}{2}$(a+c) oe), ft their (i)(a) NB $\overrightarrow{PB}=\frac{1}{2}(\mathbf{a}+\mathbf{c})$ without justification: M0A0A0E0
		$\Rightarrow \overrightarrow{OP}=\frac{1}{2}\overrightarrow{OB}$ Must see previous line	**A1***	**1.1**	$\Rightarrow \overrightarrow{PB}=\overrightarrow{OP}$
		$\Rightarrow P$ is midpt of OB or OPB is a straight line and $OP=PB$	**dep* A1**	**2.1**	
		Hence diagonals of //m bisect one another	**E1** **[4]**	**2.2a**	dep M1A1A1

20.

(a)	$k = 3$	**B1** **[1]**	**1.1**	
(b)	$(1-4)^2 + (2-k)^2 = 13$ $k = 0$ $k = 4$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1** **1.1**	oe e.g. allow consistent use of square roots – must be using subtraction in brackets
(c)	$\frac{4-2}{7-1} = \frac{k-5}{4-3}$ oe or $\frac{5-2}{3-1} = \frac{4-k}{7-4}$ oe $k = \frac{16}{3}$ $k = -\frac{1}{2}$	**M1** **A1** **[2]**	**3.1a** **1.1**	or $\frac{5-4}{3-7} = \frac{k-2}{4-1}$ oe – must be consistent application of gradients (allow one sign error) $k = \frac{5}{4}$

21.

Question	Scheme	Marks	AOs
(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y = 1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x} + 1 = -2x + 5$	M1	1.1b
	$(\times x) \Rightarrow k^2 + 1x = -2x^2 + 5x \Rightarrow 2x^2 - 4x + k^2 = 0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2 - 4ac = 0$	M1	3.1a
	$8k^2 = 16$	A1	1.1b
	$k = \pm\sqrt{2}$	A1	1.1b
		(3)	

22.

Question	Scheme	Marks	AOs
(a)	Attempts $H = mt + c$ with both $(3, 2.35)$ and $(6, 3.28)$	M1	3.3
	Method to find both m and c	dM1	3.1a
	$H = 0.31t + 1.42$ oe	A1	1.1b
		(3)	
(b)	Uses the model and states that the initial height is their 'b'	B1ft	3.4
	Compares 140 cm with their 1.42 (m) and makes a valid comment. In the case where $H = 0.31t + 1.42$ it should be this fact supports the use of the linear model as the values are close.	B1ft	3.5a
		(2)	

23.

Question	Scheme	Marks	AOs
(a)	Uses or implies that $V = ad + b$	B1	3.3
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get either a or b	M1	3.1b
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get both a and b	dM1	1.1b
	$\Rightarrow V = -\frac{1}{8}d + 50$ o.e.	A1	1.1b
		(4)	
(b)(i)(ii)	States **either** that the initial volume was 50 {litres} **or** that the distance travelled was 400 {km}	B1 ft	3.4
	Attempts to find **both** answers by solving $0 = -\frac{1}{8}d + 50$ **and** $0 = 400 - 8V$	M1	3.4
	States **both** that the initial volume was 50 litres **and** that the distance travelled was 400 km	A1	3.2b
		(3)	
(c)	States, e.g., "Poor model" as 320km is significantly less than 400 km.	B1ft	3.5a
		(1)	

24.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	

25.

(i)		$4\left[x^2-3x\right]+11$ $4\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+11$ $a=4$ $(x-3/2)^2$ $4\left(x-\frac{3}{2}\right)^2+2$ $c=2$	B1 B1 B1 [3]	1.1 1.1 1.1	No marks until attempt to complete the square Must be of the form $4(x\pm\alpha)^2\pm\ldots$
(ii)		No real roots	B1 [1]	2.2a	Zero, none, 0, … if not 'no real roots' must be consistent with their (i)
(iii)		$r=0\Rightarrow 1$ real root or 1 repeated root $r<0\Rightarrow 2$ real roots $r>0\Rightarrow$ no real roots	M1 A1 [2]	2.4 2.4	Attempt to relate the value of r to the number of real roots (this can be implied with at least one correct statement) All three statements correct

26.

Question	Scheme	Marks	AOs
(a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	