Mixed Exam Questions – Set 9

1.

(a)		$\frac{dy}{dt} = 3t^2e^{-2t} + t^3\left(-2e^{-2t}\right)$	**M1***	**2.1**	Attempts to differentiate y with respect to t using the product rule – answer of the form $\frac{dy}{dt} = \lambda t^2e^{-2t} + \mu t^3e^{-2t}$ or $y' = \alpha x^{-5}e^{-6x^{-1}}\left(\beta x + \gamma\right)$	Where $\lambda, \mu \neq 0$ Where $\alpha, \beta, \gamma \neq 0$
		$\frac{dy}{dt} = 0 \Rightarrow t^2e^{-2t}(3-2t) = 0 \Rightarrow t = \ldots$	**M1dep***	**1.1**	Sets their derivative equal to zero and solves for t	Or their $\frac{dy}{dx}$ set = 0 and solve for x
		$t = \frac{3}{2}$	**A1**	**1.1**	From correct working only (or for $x = 2$)	
		$P\left(2, \frac{27}{8}e^{-3}\right)$	**A1**	**2.2a**	From correct working only y-coordinate must be exact but ISW	
			[4]			
(b)		$\frac{dx}{dt} = -3t^{-2}$ and $\int y\frac{dx}{dt}\,dt$	**M1**	**2.1**	Differentiates x with respect to t and attempts to set up integral for the required area	With $\frac{dx}{dt} = kt^{-2}$ with non-zero k
		$x = 6 \Rightarrow t = 0.5$ and $x = 1 \Rightarrow t = 3$	**B1**	**1.1**	Stating 0.5 and 3 is sufficient for this mark	If not attempted in **(b)** then this B mark can be awarded if seen in **(c)**
		Area $= \int_3^{0.5} t^3e^{-2t}\left(-\frac{3}{t^2}\right)dt = \int_3^{0.5} -3te^{-2t}\,dt = \int_{0.5}^{3} 3te^{-2t}\,dt$	**A1**	**2.2a**	Must be correctly shown	
			[3]			

(c)		$u = 3t$, and $\mathrm{d}v$ or $\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{e}^{-2t}$	M1*	1.1	Integrating by parts as far as $\mathrm{f}(t) \pm \int \mathrm{g}(t)\,\mathrm{d}t$	Ignore limits for first three marks and allow those who consider $-\int 3t\mathrm{e}^{-2t}\mathrm{d}t$ for possibly full marks
		$\int 3t\mathrm{e}^{-2t}\,\mathrm{d}t = -\frac{3}{2}t\mathrm{e}^{-2t} + \frac{3}{2}\int \mathrm{e}^{-2t}\,\mathrm{d}t$	A1	1.1		
		$= \ldots - \frac{3}{4}\mathrm{e}^{-2t}(+c)$	A1	1.1	Allow correct un-simplified for both A marks	
		$\left[-\frac{3}{2}t\mathrm{e}^{-2t} - \frac{3}{4}\mathrm{e}^{-2t}\right]_{0.5}^{3} = \left(-\frac{3}{2}(3)\mathrm{e}^{-6} - \frac{3}{4}\mathrm{e}^{-6}\right) - \left(-\frac{3}{2}(0.5)\mathrm{e}^{-1} - \frac{3}{4}\mathrm{e}^{-1}\right)$	M1dep*	1.1	Use of their t-limits (so not 1 and 6) in fully integrated expression (must subtract bottom limit from top limit)	
		Area $= -\frac{21}{4}\mathrm{e}^{-6} + \frac{3}{2}\mathrm{e}^{-1}$	A1 [5]	2.2a	ISW once correct exact answer seen	

2.

(i)		$\mathrm{d}V/\mathrm{d}x = \pi(20x - x^2)$ $\Rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x}.\frac{\mathrm{d}x}{\mathrm{d}t}$ $= \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = k(20-x)$ $\Rightarrow \quad \pi x\frac{\mathrm{d}x}{\mathrm{d}t} = k$ *	B1 M1 A1 A1 **[4]**	 oe **ag**
(ii)		$\int \pi x\,\mathrm{d}x = \int k\,\mathrm{d}t$ $\Rightarrow \quad ½\ \pi x^2 = kt + c$ When $t = 0, x = 0 \Rightarrow c = 0$ $\Rightarrow \quad ½\ \pi x^2 = kt$ Full when $x = 10$, $t = T$ $\Rightarrow \quad 50\pi = kT$ $\Rightarrow \quad T = 50\pi/k$ *	M1 A1 B1 M1 A1 **[5]**	separate variables and attempt integration of both sides condone absence of c c=0 www substitute t or T=50 π/k or x=10 and rearranging for the other (dependent on first M1) oe **ag**, need to have c=0
(iii)		$\mathrm{d}V/\mathrm{d}t = -kx$ $\Rightarrow \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$ $\Rightarrow \pi(20-x)\frac{\mathrm{d}x}{\mathrm{d}t} = -k$*	B1 M1 A1 **[3]**	correct $\mathrm{d}V/\mathrm{d}x.\mathrm{d}x/\mathrm{d}t = \pm kx$ ft **ag**

(iv)		$\int \pi(20-x)\,dx = \int -k\,dt$	M1	separate variables and intend to integrate both sides
		$\pi(20x - \frac{1}{2}x^2) = -kt + c$	B1 A1	LHS (not dependent on M1) RHS ie $-kt + c$ (condone absence of c)
		When $t = 0$, $x = 10$ $\Rightarrow \quad \pi(200 - 50) = c$ $\Rightarrow \quad c = 150\pi$	A1	evaluation of c cao oe (x=10, t=0)
		$\Rightarrow \quad \pi(20x - \frac{1}{2}x^2) = 150\pi - kt$ $x = 0$ when $150\pi - kt = 0$	M1	substitute x=0 and rearrange for t -dependent on first M1 and non-zero c ,oe
		$\Rightarrow \quad t = 150\pi/k = 3T$*	A1 [6]	**ag**

3.

(a)	$\tan\theta = \frac{3}{4}$; bearing is 37° **(nearest degree)**	M1; A1 (2)
(b)		
(i)	$\mathbf{p} = (\mathbf{i}+\mathbf{j}) + t(2\mathbf{i}-3\mathbf{j})$	M1 A1
(ii)	$\mathbf{q} = (-2\mathbf{j}) + t(3\mathbf{i}+4\mathbf{j})$	A1
(iii)	$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (-\mathbf{i}-3\mathbf{j}) + t(\mathbf{i}+7\mathbf{j})$	M1 A1 (5)
(c)		
(i)	$-1+t=0$ $t=1$ or 3pm	M1 A1
(ii)	$-1+t=-(-3+7t)$ $t=\frac{1}{2}$ or 2.30 pm	M1 A1 (4)

4.

(i)		$a = k + 0.06t$ $k + 0.06(20) = 1.3$ $k = 1.3 - 1.2 = 0.1$	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	E E E	Use of $t = 20$ and $a = 1.3$ in their a	
(ii)			**M1***	**3.1a**	E	Attempt to integrate – all powers increased by 1 (but not just multiplying by t)	
		$s = 0.05t^2 + 0.01t^3 (+c)$	**A1ft**	**1.1**	E	$s = \frac{1}{2}kt^2 + 0.01t^3$	
		$t = 0, s = 0 \Rightarrow c = 0$	**B1**	**2.1**	A	From a correct expression for s	If $c = 0$ stated then must give a reason
		$t = 20, v = 14$	**B1ft**	**1.1**	E	$12 + 20k$	
		$s_1 = 0.05(20)^2 + 0.01(20)^3$	**dep*M1**	**3.4**	C	Finding distance travelled after 20 s (for reference $s_1 = 100$)	
		$25^2 = 14^2 + 2(1.3)s_2$	**M1**	**3.3**	A	Use of $v^2 = u^2 + 2as$ with $v = 25$ and $a = 1.3$ and their u	
		Total distance $= s_1 + s_2 = 265$ m	**A1** [7]	**2.2a**	A	All previous marks must have been awarded	

5.

i		T(before) = 0.2g = 1.96 Fr = 0.4x0.2$g$cos30 (=0.67896..) 0.2a = 0.2g – T Either correct 0.2a =T - 0.2$g$sin30 - 0.4x0.2$g$cos30 Both correct 2T = 0.2g + 0.2$g$sin30 + 0.4x0.2$g$cos30 T = 1.81 N	**B1** **B1** **M1** **A1** **M1** **A1** **[6]**
ii		THIS CANNOT BE SOLVED USING a(i) 0.2a = +/- (0.2$g$sin30 + 0.4x0.2$g$cos30) a = +/-(8.2948..) v^2 = 2x8.29(48..)x0.8 *OR* 0 = u^2 - 2x8.29(48..)x0.8 v = 3.64 m s^{-1} or u = 3.64 m s^{-1}	**M1*** **A1** **A1** **D*M1** **A1** **[5]**
iii		 $R^2 = (0.2g\cos30)^2 + (0.4x0.2g\cos30)^2$ R=1.83 N	**M1** **A1** **A1** **[3]**