Mixed Exam Questions – Set 8

1.

(a)	**(i)**	$a = 2$	**B1**	**1.1**	Either stated or embedded in equation	eg $\|2x - b\|$ seen ignore any other values seen **B0** for $a = -2$, unless subsequently corrected
		$b = 6$	**B1**	**1.1**	Either stated or embedded in equation	eg $\|ax - 6\|$ seen ignore any other values seen
		$c = 1$	**B1** **[3]**	**1.1**	Either stated or embedded in equation	eg $\|ax - b\| + 1$ seen ignore any other values seen
(a)	**(ii)**	Because f is a many to one function eg f(0) = f(6)	**B1** **[1]**	**1.2**	Any correct reason	Condone no explicit example Could also say 'because f is not one to one' **B1** BOD for 'it is not one to one' If referring to 'one to many' or 'many to one' it must be clear whether this is f or f^{-1} (just 'it' or 'the function' is not enough) Allow implication of function eg 'as it is a many to one function there is no inverse function' May also refer to the 'horizontal line test', but need to state outcome eg horizontal line would cross graph of $y = f(x)$ twice
(b)	**(i)**	$y = px - q$ $px = y + q$ $x = \frac{1}{p}(y + q)$	**M1**	**3.1a**	Complete attempt to find inverse function of $f(x) = px - q$	Correct order of operations, allow sign error only Could use coordinate geometry and reflection in $y = x$ Allow **M1** BOD if more than one function is being considered
		$g^{-1}(x) = \frac{1}{p}x + \frac{q}{p}$	**A1**	**1.1**	Obtain correct inverse, in terms of x	Could be single term ie $g^{-1}(x) = \frac{x+q}{p}$ **A1** for just $\frac{1}{p}x + \frac{q}{p}$, ie $g^{-1}(x)$ can be omitted If LHS seen, it must be $g^{-1}(x)$ or y (allow BOD for g^{-1}, or using f not g) BOD if modulus sign included **A0** if additional equations given
		$x \geq 0$	**B1** **[3]**	**1.2**	Correct domain **B0** for $x > 0$	Independent of the first two marks If in words then must be correct, so **B1** for 'any non-negative x' but **B0** for 'any positive x' $g^{-1}(x) \geq 0$ is **B0** Condone incorrect set notation as long as intention is clear
(b)	**(ii)**	$0 < p \leq 1$	**B1**	**3.1a**	Correct set of values, any notation No need for $0 < p$ as specified in question, so **B1** for $p \leq 1$	**B0** for $p < 1$ **B0** for any additional incorrect values **B0** if just single example and not set of values Condone incorrect set notation as long as intention is clear

2.

(a)	**(i)**	$f'(x) = -\sin x + \sqrt{3}\cos x$	**M1**	**3.1a**	Attempt differentiate f(*x*), allow sign errors but both trig functions must be changed. oe e.g. $2\cos\left(x + \frac{\pi}{6}\right)$
		$\tan x = \sqrt{3}$	**M1**	**1.1**	Setting their $f'(x) = 0$ and correctly manipulating to reach an equation in a single trig function (e.g. $4\cos^2 x = 1$ or $4\sin^2 x = 3$) do not allow incorrect working e.g. $a - b = 0 \Rightarrow a^2 - b^2 = 0$
		$x = \frac{1}{3}\pi$	**A1**	**1.1**	www, must be in radians (allow decimal 1.05 3sf)
		or $\frac{4}{3}\pi$	**A1** **[4]**	**1.1**	www, must be in radians (allow decimal 4.19 3sf), and for this mark have no other solutions in the given range (ignore any outside) SC**B1** for each correct solution (max [2/4]) if insufficient or no working shown, but not from incorrect working. If both solutions given correctly in degrees (60°,240°) then can get M1M1SCB1 (max [3/4])
(a)	**(ii)**	$f''(x) = -\cos x - \sqrt{3}\sin x$ or…	**M1**	**3.1a**	Attempt to differentiate their f' (x) (allow sign errors but both trig functions must be changed), setting their $f''(x) = 0$ and attempting to manipulate. May see $f''(x) = -2\sin\left(x + \frac{\pi}{6}\right)$
		$\tan x = -\frac{1}{\sqrt{3}}$	**A1**	**1.1**	For correctly reaching an equation in a single trig function (oe, may see $4\sin^2 x = 1$ or $4\cos^2 x = 3$) Not from incorrect working e.g. $a - b = 0 \Rightarrow a^2 - b^2 = 0$
		$x = \frac{5}{6}\pi$ or $x = \frac{11}{6}\pi$	**A1** **[3]**	**1.1**	www, for both correct in radians (allow decimals 2.62, 5.76 3sf) SC**B1** for both correct solutions (max [1/3]) if insufficient or no working shown, but not from incorrect working. Answers in degrees can get M1A1A0 (max [2/3])
(b)		A: $(\frac{1}{3}\pi, 0)$ AND C: $(\frac{4}{3}\pi, 0)$	**B1FT**	**1.1**	FT their *x*-values in radians or degrees from **(a)(i)**, both with *y*=0 Both must be correctly labelled with *A* and *C*, allow decimals 3sf
		B: $(\frac{5}{6}\pi, -2)$ AND D: $(\frac{11}{6}\pi, 2)$	**B1FT** **[2]**	**1.1**	FT their *x*-values in radians or degrees from **(a)(ii),** with both *y*=-2 and *y*=2 correct Both must be correctly labelled with *B* and *D*, allow decimals 3sf If neither mark awarded then can get one of (max [1/2]): SC**B1FT** for any two correct pairs *x* and *y*, correctly labelled SC**B1FT** for all four *x*-coords, correctly labelled and in ascending order (only if 2 distinct answers given in both **(a)(i)** and **(a)(ii)**) SC**B1FT** for all four *y*-coords, correctly labelled
(c)	**(i)**	Where the graph (of f '(*x*)) is above the *x*-axis or the graph is positive	**B1** **[1]**	**2.4**	Must reference the graph – do not accept just 'gradient is positive' or just 'f'(x)>0'
(c)	**(ii)**	$0 \le x < \frac{1}{3}\pi$, $\frac{4}{3}\pi < x \le 2\pi$	**B1**	**2.2a**	Both intervals correctly identified, ignore set notation for this mark cao (values must be correct) but condone any clear indication of both correct intervals e.g. $0 \to \frac{\pi}{3}$ and $\frac{4\pi}{3} \to 2\pi$ (allow decimals) Accept any combination of ≤ and < or ( ) and [ ]
		$\{x: 0 \le x < \frac{1}{3}\pi\} \cup \{x: \frac{4}{3}\pi < x \le 2\pi\}$ Or $[0, \frac{1}{3}\pi) \cup (\frac{4}{3}\pi, 2\pi]$	**B1** **[2]**	**2.5**	Writing their answer in correct set notation, values may be incorrect for this mark but there must be two separate intervals. Allow single-tailed inequalities as long as written in correct set notation, e.g. $\left\{x: x < \frac{\pi}{3}\right\} \cup \left\{x: x > \frac{4\pi}{3}\right\}$ Accept any combination of ≤ and < or ( ) and [ ] but do not accept ∩ instead of ∪.

3.

(a)		$a^2 = 4b + 2$	**M1**	**2.1**	Setting up so that the deduction a^2 is even can be made.
		Hence a^2 is even. Hence a is even	**A1**	**2.2a**	www, must see both statements and a convincing, correct, argument oe (e.g. $a^2 = 2(2b+1)$)
		Alternative method Assume that a is odd, then a^2 is odd	**M1**		For setting up and stating that a is odd $\Rightarrow a^2$ is odd May see (not required) $a = 2n+1$, $a^2 = 2(2n^2+2n)+1$ Hence a^2 is odd
		$4b$ is even, so $a^2 - 4b$ is odd Hence 2 is odd (so contradiction) Hence a is even.	**A1** **[2]**		www, Must see both statements and a convincing, correct, argument
(b)		Assume that $a^2 - 4b = 2$ Let $a = 2n$, (where n is an integer)	**M1**	**2.1**	Setting up (must see assumption and use of a is even)
		Either of: $4n^2 - 4b = 2$, $2n^2 - 2b = 1$, | $4n^2 - 4b = 2$, $n^2 - b = 0.5$,	**A1**	**2.1**	Substituting in $a = 2n$ and correctly reaching an equation which shows a contradiction. Accept the equivalent in words if clear and correct. Also accept: $4n^2 - 4b$ is a multiple of 4, Hence $a^2 - 4b$ is a multiple of 4, which is a contradiction
		Hence 1 is even | $n^2 - b$ is an integer (Contradiction) Hence $a^2 - 4b \neq 2$	**A1**	**2.2a**	www, Must see both statements and a convincing, correct, argument

4.

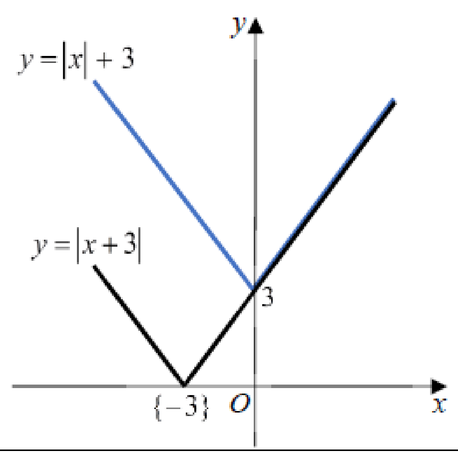
(a)		$B(20, \frac{1}{3})$	**B1** **[1]**	**3.3**	Allow '$n = 20, p = \frac{1}{3}$' or just $(20, \frac{1}{3})$. Accept $p = 0.\dot{3}$ but not 0.333
(b)	**(i)**	${}^{20}C_x(\frac{2}{3})^{(20-x)}(\frac{1}{3})^x$	**B1FT** **[1]**	**3.4**	FT their values of n and p and allow $\binom{20}{x}$ as alternative to ${}^{20}C_x$ (If decimals used both must be correct to 3sf)
(b)	**(ii)**	$\{x : x \in \mathbb{Z}, 0 \le x \le 20\}$	**B1FT** **[1]**	**1.2**	FT their '20' only Any acceptable alternative in set notation Indication that x is an integer must be present (but condone 'integer' or 'whole number' or $x \in \mathbb{N}$)
(c)		$P(X \le 9) - P(X \le 4)$ attempted	**M1**	**3.4**	Condone $P(X \le 9) - P(X \le 5)$, or $0.90810 - 0.29721$ or 0.611 May be implied by correct working or answer. Allow sum of $P(X = x)$ for x from 5 to 9.
		$(= 0.90810 - 0.15151)$ $= 0.757$ (3 sf)	**A1** **[2]**	**1.1**	**BC** cao
(d)		May include too many from one year and too few from another year	**B1** **[1]**	**3.5b**	For an explanation (which must be in context) showing clear recognition that a random sample (of this size) would not be representative across the year groups. Acceptable similar answers in context include: • 'should include roughly equal numbers from each year' • 'not representative of each year group' • 'may not include students from every year group' Do not accept 'may end up with more students from Y7-11 than from Y12-13' (because a representative sample would have this) Must be in context – so do not accept generic answers without context such as just: 'It might be biased' or 'It's not representative' or 'it should be a stratified sample' Do not accept answers solely referring to the suitability of the individual students within the sample to represent the school

5.

(a)		P(has disease | positive result) $= \dfrac{\text{P(has disease \& positive result)}}{\text{P(positive result)}}$	M1	3.4	Attempting this calculation, allow wrong values but for this mark must be a fraction with a product in the numerator and a sum of two products in the denominator.
		$= \dfrac{0.35\times0.95}{0.35\times0.95+0.65\times0.1}$	A1	1.1	Fully correct expression
		$= 0.836$ (3 sf)	A1 [3]	1.1	Or 133/159 or 0.8365 (4sf) (0.836477…)
(b)		(Let proportion having the disease = p) $p \times 0.95 + (1-p)\times 0.1$	M1	1.1	Setting up an expression in this form using the given values
		$p \times 0.95 + (1-p)\times 0.1 = 0.43$	M1	3.4	Setting their expression =0.43 and attempting to solve
		$0.85p = 0.33$ $p = 0.388$	A1	1.1	cao (watch for 0.389 from incorrect working)
		About 39% of trees (in county *B*) have the disease	B1FT [4]	3.2a	"Around 38.8 or 39 or 40" (oe e.g. 2/5). Must be in context and include "about" or "approximately" or "roughly" oe

6.
(a)

Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."	
E.g. $m = \sqrt{3},\ n = \sqrt{12}$	M1
$\{mn =\}\ (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational **or** 6 is rational	A1
	(2)

(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1
		Superimposes the graph of $y = \|x+3\|$ on top of the graph of $y = \|x\| + 3$	M1
	the graph of $y = \|x\| + 3$ is either the same or above the graph of $y = \|x+3\|$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \|x\| + 3$ is above the graph of $y = \|x+3\|$		A1

7.

$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$; as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$		
$\frac{\cos(\theta+h) - \cos\theta}{h}$	B1	2.1
$= \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$	M1 A1	1.1b 1.1b
$= -\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta$		
As $h \to 0$, $-\frac{\sin h}{h}\sin\theta + \left(\frac{\cos h - 1}{h}\right)\cos\theta \to -1\sin\theta + 0\cos\theta$	dM1	2.1
so $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ *	A1*	2.5

8.

(a)	$\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$ (for k or a numerical k)		M1
	$\int r^2\,dr = \int \pm k\,dt \Rightarrow ...$ (for k or a numerical k)		M1
	$\frac{1}{3}r^3 = \pm kt\ \{+c\}$		A1
	$t=0, r=5$ and $t=4, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{6}t + \frac{125}{3}$,	$t=0, r=5$ and $t=240, r=3$ gives $\frac{1}{3}r^3 = -\frac{49}{360}t + \frac{125}{3}$,	M1
	where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth	where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	A1
(b)	$r = 0 \Rightarrow 0 = -\frac{49}{6}t + \frac{125}{3} \Rightarrow 0 = -49t + 250 \Rightarrow t = ...$		M1
	time = 5 minutes 6 seconds		A1
			(2)
(c)	Suggests a suitable limitation of the model. E.g. • Model does not consider how the mint is sucked • Model does not consider whether the mint is bitten • Model is limited for times up to 5 minutes 6 seconds, o.e. • Not valid for times greater than 5 minutes 6 seconds, o.e. • Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked • The model indicates that the radius of the mint is negative after it dissolves • Model does not consider the temperature in the mouth • Model does not consider rate of saliva production • Mint could be swallowed before it dissolves in the mouth		B1
			(1)

9.

(a)

$N=\dfrac{900}{3+7e^{-0.25t}}=900(3+7e^{-0.25t})^{-1}, t\in\mathbb{R}, t\geq 0$; $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$	
90	B1
	(1)

(b) Way 1	$\dfrac{dN}{dt}=-900(3+7e^{-0.25t})^{-2}\left(7(-0.25)e^{-0.25t}\right)$ $\left\{=\dfrac{900(0.25)(7)e^{-0.25t}}{(3+7e^{-0.25t})^2}\right\}$	M1 A1
	$\Rightarrow \dfrac{dN}{dt}=\dfrac{900(0.25)\left(\left(\dfrac{900}{N}-3\right)\right)}{\left(\dfrac{900}{N}\right)^2}$	dM1
	correct algebra leading to $\dfrac{dN}{dt}=\dfrac{N(300-N)}{1200}$ *	A1*
		(4)
(c)	Deduces $N=150$ (can be implied)	B1
	so $150=\dfrac{900}{3+7e^{-0.25T}} \Rightarrow e^{-0.25T}=\dfrac{3}{7}$	M1
	$T=-4\ln\left(\dfrac{3}{7}\right)$ or $T=$ awrt 3.4 (months)	dM1 A1
		(4)
(d)	either one of 299 or 300	B1
		(1)

10.

(a)		$AB=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$	**B1** **[1]**	**1.1**	Correct length aef	Condone 4.47 or better Allow isw eg $\sqrt{20}=4\sqrt{5}$ Allow BOD on signs eg $AB=-2\mathbf{i}-4\mathbf{j}$ seen
(b)		$(p-3)^2+(p-5)^2=20$	**M1**	**1.1a**	Attempt correct equation for length *BC*	Using their attempt at length of *AB* Condone error on RHS eg having $\sqrt{20}$ not 20
		$p^2-8p+7=0$ $p=7$	**A1**	**1.1**	**BC** Solve correct quadratic to obtain at least $p=7$	If second value of p stated then it must be correct
		C is $7\mathbf{i}+7\mathbf{j}$	**A1**	**1.1**	Correct position vector for *C*; it could be given as column vector, but not coordinate	No need to discard $p=1$
			[3]			If **M0**, question is 'determine' so some evidence needed for full marks – either justifying lengths are equal, or use of components of 2 and 4 $7\mathbf{i}+7\mathbf{j}$ with some explanation **B3** $7\mathbf{i}+7\mathbf{j}$ with no explanation **B2** (7, 7) with some explanation **B2** (7, 7) with no explanation **B1**

(c)		*OM* is 4**i** + 4**j** OR *BM* is **i** – **j**	**B1**	**1.1**	Correct midpoint soi Could instead find vector *BM*	Allow *M* seen as coordinate, as it is part of their method and not a requested answer Condone *M* = 4**i** + 4**j**, but penalise clear error eg *AM* = 4**i** + 4**j** is **B0** Could be soi on a diagram
		D is 6**i** + 2**j**	**B1**	**1.1**	Correct position vector (not coordinate) for *D*	Do not penalise *D* given as coordinate if already penalised in part **(b)**
			[2]			Answer only is **B0B1**
(d)		Kite	**B1***	**2.2a**	Mark independently of reason	
			B1dep*	**2.2a**	Evidence is required to support statements made	All relevant evidence quoted must be correct
		eg two pairs of adjacent sides of same length			$AD = CD = \sqrt{26}$ (or compare components of vectors) ; condone not stating $AB = BC$ as given in question	Sides must be defined as adjacent, so **B0** for just 'two pairs of equal sides', but allow BOD if clarified on an explicit diagram seen in part **(d)**
		eg diagonals are perpendicular			*AC* has gradient of 1, *BD* has gradient of –1	If using a geometrical argument, then identify that *ABC* is isosceles, *M* is mid-point of *AC* hence perpendicular bisector
		eg *BD* being a line of symmetry			*AM* = *MC*, with perpendicular argument as above	
					B0 for reasoning using angles (ie a pair of facing equal angles) unless justified.	
			[2]			