Mixed Exam Questions – Set 7

1.

Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

2.

The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

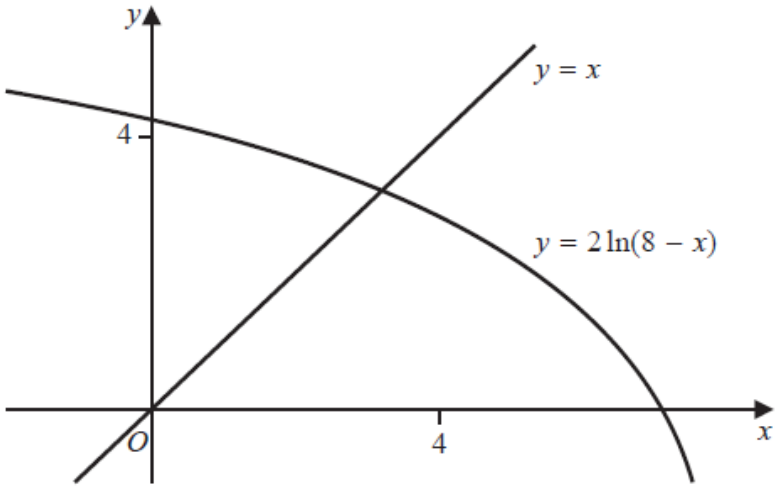

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

3.

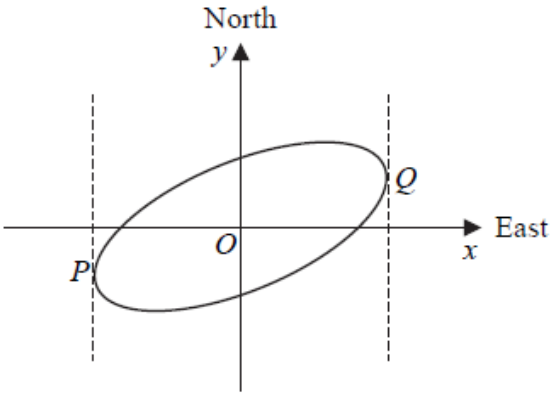

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\frac{dy}{dx} = \frac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

4.

(i) Show that $\sum_{r=1}^{16} (3 + 5r + 2^r) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

5.

The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

6.

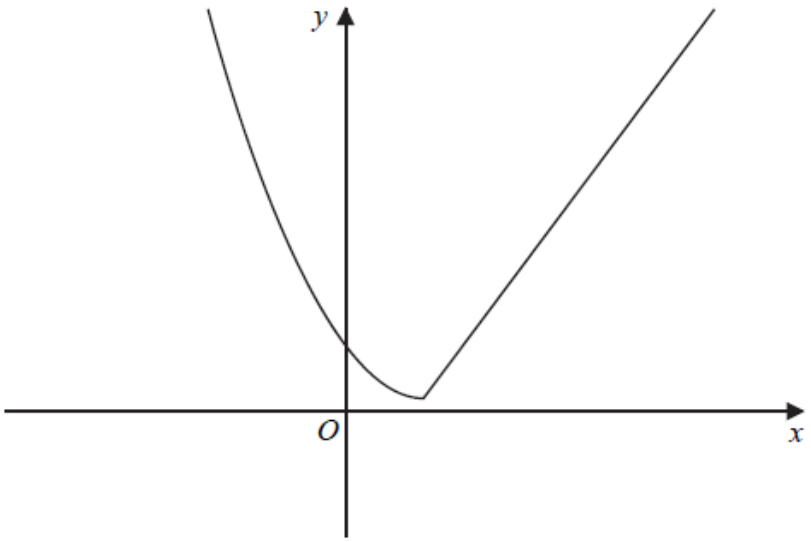

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $\mathrm{gg}(0)$.

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x - 2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)