Mixed Exam Questions – Set 6

1.

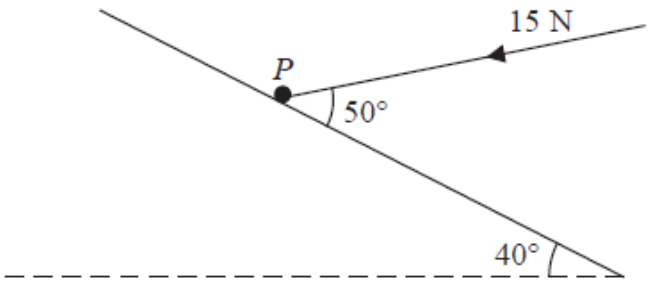


Figure 4

A particle P of mass 2.7 kg lies on a rough plane inclined at 40° to the horizontal. The particle is held in equilibrium by a force of magnitude 15 N acting at an angle of 50° to the plane, as shown in Figure 4. The force acts in a vertical plane containing a line of greatest slope of the plane. The particle is in equilibrium and is on the point of sliding down the plane.

Find

(a) the magnitude of the normal reaction of the plane on P,

(4)

(b) the coefficient of friction between P and the plane.

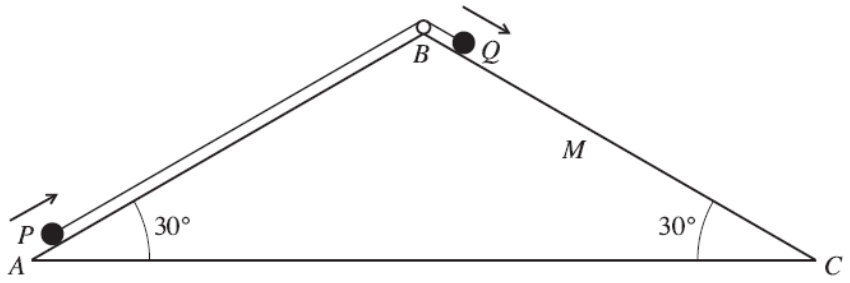
(5)

The force of magnitude 15 N is removed.

(c) Determine whether P moves, justifying your answer.

(4)

2.



AB and *BC* are lines of greatest slope on a fixed triangular prism, and *M* is the mid-point of *BC*. *AB* and *BC* are inclined at 30° to the horizontal. The surface of the prism is smooth between *A* and *B*, and between *B* and *M*. Between *M* and *C* the surface of the prism is rough. A small smooth pulley is fixed to the prism at *B*. A light inextensible string passes over the pulley. Particle *P* of mass 0.3 kg is fixed to one end of the string, and is placed at *A*. Particle *Q* of mass 0.4 kg is fixed to the other end of the string and is placed next to the pulley on *BC*. The particles are released from rest with the string taut. *P* begins to move towards the pulley, and *Q* begins to move towards *M* (see diagram).

(i) Show that the initial acceleration of the particles is $0.7\,\text{m s}^{-2}$, and find the tension in the string. **[5]**

The particle *Q* reaches *M* 1.8 s after being released from rest.

(ii) Find the speed of the particles when *Q* reaches *M*. **[2]**

After *Q* passes through *M*, the string remains taut and the particles decelerate uniformly. *Q* comes to rest between *M* and *C* 1.4 s after passing through *M*.

(iii) Find the deceleration of the particles while *Q* is moving from *M* towards *C*. **[2]**

(iv) **(a)** By considering the motion of *P*, find the tension in the string while *Q* is moving from *M* towards *C*. **[3]**

(b) Calculate the magnitude of the frictional force which acts on *Q* while it is moving from *M* towards *C*. **[3]**

3.

Mr McGregor is a keen vegetable gardener. A pigeon that eats his vegetables is his great enemy.

One day he sees the pigeon sitting on a small branch of a tree. He takes a stone from the ground and throws it. The trajectory of the stone is in a vertical plane that contains the pigeon. The same vertical plane intersects the window of his house. The situation is illustrated in Fig. 5.

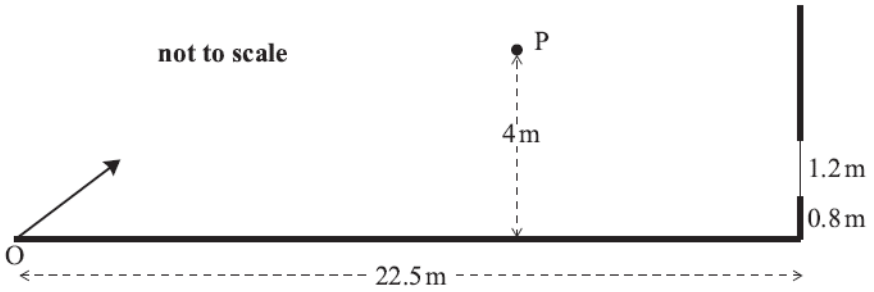


Fig. 5

- The stone is thrown from point O on level ground. Its initial velocity is $15\,\text{m s}^{-1}$ in the horizontal direction and $8\,\text{m s}^{-1}$ in the vertical direction.
- The pigeon is at point P which is 4 m above the ground.
- The house is 22.5 m from O.
- The bottom of the window is 0.8 m above the ground and the window is 1.2 m high.

Show that the stone does not reach the height of the pigeon.

Determine whether the stone hits the window. **[7]**

4.

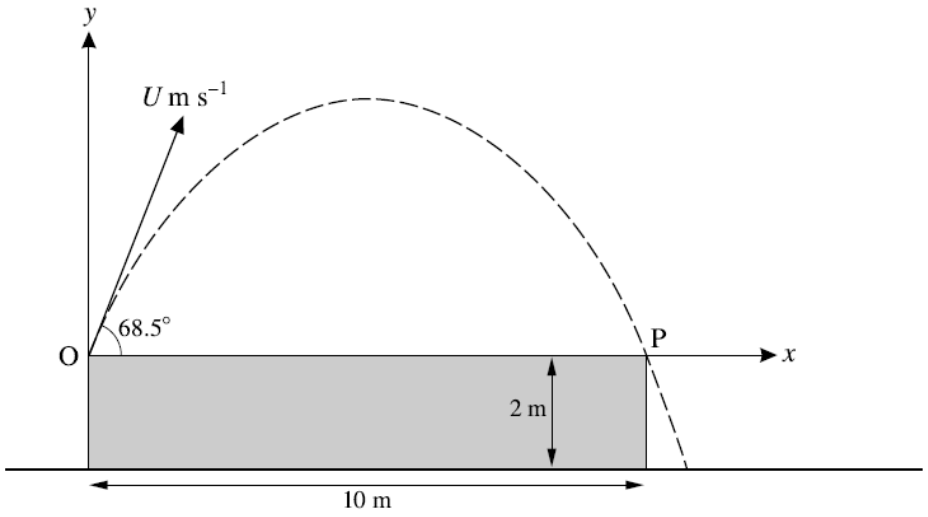


Fig. 7

Fig. 7 shows a platform 10 m long and 2 m high standing on horizontal ground. A small ball projected from the surface of the platform at one end, O, just misses the other end, P. The ball is projected at $68.5°$ to the horizontal with a speed of $U\,\text{m s}^{-1}$. Air resistance may be neglected.

At time t seconds after projection, the horizontal and vertical displacements of the ball from O are x m and y m.

(i) Obtain expressions, in terms of U and t, for

(A) x,

(B) y. **[3]**

(ii) The ball takes T s to travel from O to P.

Show that $T = \dfrac{U \sin 68.5°}{4.9}$ and write down a second equation connecting U and T. **[4]**

(iii) Hence show that $U = 12.0$ (correct to three significant figures). **[3]**

(iv) Calculate the horizontal distance of the ball from the platform when the ball lands on the ground. **[5]**

(v) Use the expressions you found in part **(i)** to show that the cartesian equation of the trajectory of the ball in terms of U is

$$y = x\tan 68.5^\circ - \frac{4.9x^2}{U^2(\cos 68.5^\circ)^2}.$$

Use this equation to show again that $U = 12.0$ (correct to three significant figures). **[4]**

5.

A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of d,

(ii) the value of M.

(7)

6.

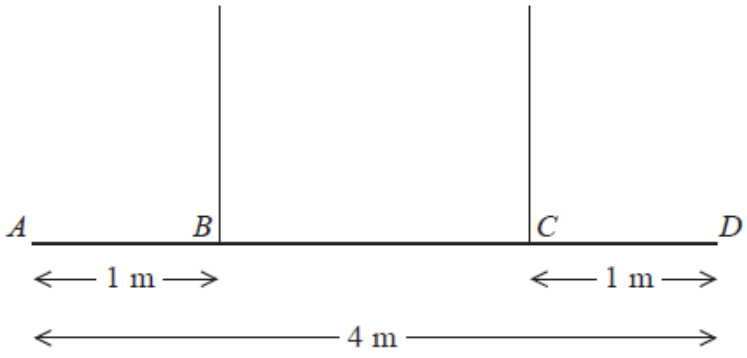


Figure 3

A non-uniform beam AD has weight W newtons and length 4 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. The ropes are attached to two points B and C on the beam, where $AB = 1$ m and $CD = 1$ m, as shown in Figure 3. The tension in the rope attached to C is double the tension in the rope attached to B. The beam is modelled as a rod and the ropes are modelled as light inextensible strings.

(a) Find the distance of the centre of mass of the beam from A.

(6)

A small load of weight kW newtons is attached to the beam at D. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle.

Find

(b) an expression for the tension in the rope attached to B, giving your answer in terms of k and W,

(3)

(c) the set of possible values of k for which both ropes remain taut.

(2)