Mixed Exam Questions – Set 5

1.

(a) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(3)

2.

(a) Find the binomial expansion of $(1+5x)^{\frac{1}{5}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(8+3x)^{-\frac{2}{3}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an estimate for $\sqrt[3]{\frac{1}{81}}$, giving your answer to four decimal places.

[2 marks]

3.

(a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}$, $|x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots$$

(b) find the value of the constant k,

(2)

(c) find the value of the constant A.

(2)

4.

$$f(x) = \frac{6}{\sqrt{(9-4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{6}{\sqrt{(9+4x)}}, \quad |x| < \frac{9}{4}$

(1)

(c) $h(x) = \frac{6}{\sqrt{(9-8x)}}, \quad |x| < \frac{9}{8}$

(2)

5.

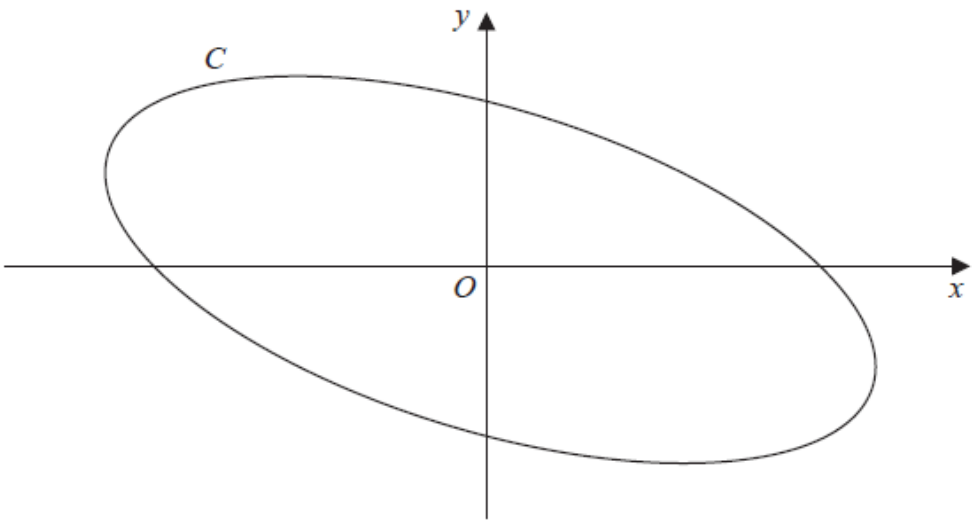


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leqslant t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

6.

P is a general point on the curve with parametric equations $x = 2t$, $y = \frac{2}{t}$. This is shown in Fig. 6. The tangent at P intersects the x- and y-axes at the points Q and R respectively.

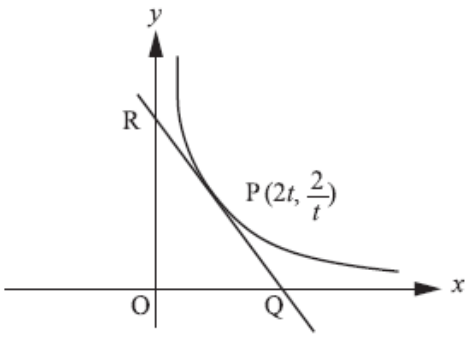


Fig. 6

Show that the area of the triangle OQR, where O is the origin, is independent of t. **[7]**

7.

The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

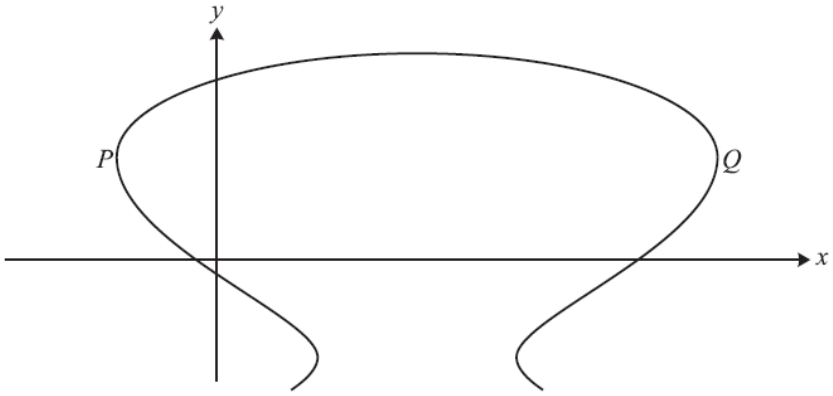
(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

8.



The diagram shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$. At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. **[8]**