Answers - Mixed Exam Questions – Set 5

1.

(a)	$\left\{\sqrt[3]{(8-9x)}\right\} = (8-9x)^{\frac{1}{3}}$	Power of $\frac{1}{3}$	M1
	$= \underline{(8)^{\frac{1}{3}}}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$	$\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	$\underline{\text{B1}}$
	$=\{2\}\left[1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$=\{2\}\left[1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots\right]$		
	$= 2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$	See notes below!	
	$= 2-\frac{3}{4}x;-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$		A1; A1
			[6]
(b)	$\left\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\right\}$ so $x=0.1$	Writes down or uses $x=0.1$	B1
	When $x=0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+\ldots$		M1
	$= 2-0.075-0.0028125-0.00017578125$		
	$= 1.922011719$		
	So, $\sqrt[3]{7100} = 19.220117919\ldots = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**
			[3]

2.

(a)	$1+\frac{1}{5}\times 5x+kx^2$	M1		k any non-zero numerical expression
	$1+x-2x^2$	A1	2	Simplified to this
(b) (i)	$(8+3x)^{-\frac{2}{3}} = 8^{-\frac{2}{3}}(1+\frac{3}{8}x)^{-\frac{2}{3}}$	B1		ACF for $8^{-\frac{2}{3}} = \frac{1}{4}$
	$\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$ $=1+\left(-\frac{2}{3}\right)\left(\frac{3}{8}x\right)+\frac{1}{2}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{3}{8}x\right)^2$	M1		Expand correctly using their $\frac{3}{8}x$ Condone poor use of or missing brackets
	$\frac{1}{4}-\frac{1}{16}x+\frac{5}{256}x^2$	A1	3	Accept $=\frac{1}{4}\left(1-\frac{1}{4}x+\frac{5}{64}x^2\right)$
(ii)	$x=\frac{1}{3}$	M1		$x=\frac{1}{3}$ **used** in their expansion from (b)(i)
	0.2313 (4dp)	A1	2	Note **3** in **4th** decimal place

3.

(a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$	<u>B1</u>
	$=\left\{\frac{1}{4}\right\}\left[1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2+\dots\right]$	see notes	M1 A1ft
	$=\left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x}{2}\right)^2+\dots}\right]$		
	$=\frac{1}{4}\left[1+5x;+\frac{75}{4}x^2+\dots\right]$	See notes below!	
	$=\frac{1}{4}+\frac{5}{4}x;+\frac{75}{16}x^2+\dots$		A1; A1
			[5]
(b)	$\left\{\frac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\left\{\frac{75}{16}x^2+\dots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\frac{2(5x)}{4}+\frac{kx}{4}=\frac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\frac{150x^2}{16}+\frac{5kx^2}{4}$		M1
	So, $A=\frac{75}{8}+\frac{5(-3)}{4} = \frac{75}{8}-\frac{15}{4} = \underline{\frac{45}{8}}$	$\underline{\frac{45}{8}}$ or $\underline{5\frac{5}{8}}$ or $\underline{5.625}$	A1
			[2] 9

4.

(a) $\mathrm{f}(x) = \dots(\dots - \dots x)^{-\frac{1}{2}}$		M1	
$= 6\times 9^{-\frac{1}{2}}(\dots)$	$\frac{6}{9^{\frac{1}{2}}}$, $\frac{6}{3}$, 2 or equivalent	B1	
$= \dots\left(1+(-\tfrac{1}{2})(kx);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(kx)^3+\dots\right)$		M1; A1ft	
$=2\left(1+\frac{2}{9}x+\dots\right)$	or $2+\frac{4}{9}x$	A1	
$=2+\frac{4}{9}x+\frac{4}{27}x^2+\frac{40}{729}x^3+\dots$		A1	**(6)**
(b) $\mathrm{g}(x)=2-\frac{4}{9}x+\frac{4}{27}x^2-\frac{40}{729}x^3+\dots$		B1ft	**(1)**
(c) $\mathrm{h}(x)=2+\frac{4}{9}(2x)+\frac{4}{27}(2x)^2+\frac{40}{729}(2x)^3+\dots$		M1 A1	**(2)**
$\left(=2+\frac{8}{9}x+\frac{16}{27}x^2+\frac{320}{729}x^3+\dots\right)$			**[9]**

5.

(a)	**Main Scheme**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x+y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**Main Scheme**		
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	A1
			[2]
(b)	**Alternative Method 1**		
	$(x+y)^2 = 12\cos^2 t = 12(1-\sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x+y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x+y)^2 + 3y^2 = 12$		M1, A1
			[2]

6.

$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-2/t^2}{2}\left(=-\frac{1}{t^2}\right)$	M1* A1 M1dep*
$y-\frac{2}{t}=-\frac{1}{t^2}(x-2t)$	A1
When $x=0,\ y=\frac{4}{t}$	A1ft
When $y=0, x=4t$	A1ft
So area of triangle $=\frac{1}{2}\times\frac{4}{t}\times 4t=8$ (which is independent of t)	A1 [7]
OR (for the first two marks) $y=\frac{2}{\left(\frac{x}{2}\right)}=\frac{4}{x}\Rightarrow\frac{dy}{dx}=-\frac{4}{x^2}$	M1*
$\Rightarrow\left(\frac{dy}{dx}\right)=-\frac{4}{(2t)^2}$	A1

7.

(a)

$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}\ \underline{8x-3y^2\frac{dy}{dx}}\ \underline{\underline{-4y-4x\frac{dy}{dx}}}\ +\overline{\overline{2^y\ln 2\frac{dy}{dx}}}=\underline{0}$		M1 A1 M1 B1
$8(-2)-3(4)^2\frac{dy}{dx}-4(4)-4(-2)\frac{dy}{dx}+2^4\ln 2\frac{dy}{dx}=0$	**dependent on the first M mark**	dM1
$-16-48\frac{dy}{dx}-16+8\frac{dy}{dx}+16\ln 2\frac{dy}{dx}=0$		
$\frac{dy}{dx}=\frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent		A1 cso

(b)

e.g. $m_N = \dfrac{-40+16\ln 2}{-32}$ or $\dfrac{40-16\ln 2}{32}$	Applying $m_N = \dfrac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
• $y - 4 = \left(\dfrac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\dfrac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\dfrac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N ($\square m_T$), either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
$\left\{\Rightarrow c = 4 + \dfrac{40-16\ln 2}{16}, \text{ so } y = \dfrac{104-16\ln 2}{16} \Rightarrow\right\}$		
y (or c) $= \dfrac{13}{2} - \ln 2$	$\dfrac{104}{16} - \ln 2$ or $\dfrac{13}{2} - \ln 2$ or $-\ln 2 + \dfrac{13}{2}$	A1 cso isw
Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		[3]

8.

$3y^2 \dfrac{dy}{dx}$	B1	or $2x\dfrac{dx}{dy}$
$2x - 12\dfrac{dy}{dx} - 8$	B1	$3y^2 - 8\dfrac{dx}{dy} - 12$
their $3y^2\dfrac{dy}{dx} - 12\dfrac{dy}{dx} = 8 - 2x$ soi must be two terms on each side and must follow from RHS $= 0$	M1	their $2x\dfrac{dx}{dy} - 8\dfrac{dx}{dy} = -3y^2 + 12$ must be two terms on each side must follow from RHS $= 0$
$\dfrac{dy}{dx} = \dfrac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\dfrac{dx}{dy} = 0$ is substituted and there is no evidence for an incorrect expression for $\dfrac{dx}{dy}$
their $3y^2 - 12 = 0$	M1*	
$y = (\pm)\,2$	A1	A0 if $\dfrac{dy}{dx}$ incorrect
substitution of their positive y value in original equation	M1dep*	
$x = 10, x = -2$ and no others cao	A1	A0 if $\dfrac{dy}{dx}$ incorrect