Roots of Polynomial Equations

Exercise A

1 α and β are the roots of the quadratic equation $3x^2 + 7x - 4 = 0$. Without solving the equation, find the values of:

a $\alpha + \beta$ **b** $\alpha\beta$ **c** $\frac{1}{\alpha} + \frac{1}{\beta}$ **d** $\alpha^2 + \beta^2$

2 α and β are the roots of the quadratic equation $7x^2 - 3x + 1 = 0$. Without solving the equation, find the values of:

a $\alpha + \beta$ **b** $\alpha\beta$ **c** $\frac{1}{\alpha} + \frac{1}{\beta}$ **d** $\alpha^2 + \beta^2$

3 α and β are the roots of the quadratic equation $6x^2 - 9x + 2 = 0$. Without solving the equation, find the values of:

a $\alpha + \beta$ **b** $\alpha^2 \times \beta^2$

c $\frac{1}{\alpha} + \frac{1}{\beta}$ **d** $\alpha^3 + \beta^3$

Hint Try expanding $(\alpha + \beta)^3$.

4 The roots of a quadratic equation $ax^2 + bx + c = 0$ are $\alpha = 2$ and $\beta = -3$.
Find integer values for a, b and c.

5 The roots of a quadratic equation $ax^2 + bx + c = 0$ are $\alpha = -\frac{1}{2}$ and $\beta = -\frac{1}{3}$
Find integer values for a, b and c.

6 The roots of a quadratic equation $ax^2 + bx + c = 0$ are $\alpha = \frac{-1 + i}{2}$ and $\beta = \frac{-1 - i}{2}$
Find integer values for a, b and c.

7 One of the roots of the quadratic equation $ax^2 + bx + c = 0$ is $\alpha = -1 - 4i$.

a Write down the other root, β.

b Given that $a = 1$, find the values of b and c.

8 Given that $kx^2 + (k - 3)x - 2 = 0$, find the value of k if the sum of the roots is 4.

9 The equation $nx^2 - (16 + n)x + 256 = 0$ has real roots α and $-\alpha$. Find the value of n.

10 The roots of the equation $6x^2 + 36x + k = 0$ are reciprocals of each other. Find the value of k.

11 The equation $mx^2 + 4x + 4m = 0$ has roots of the form k and $2k$. Find the values of m and k.

12 The equation $ax^2 + 8x + c = 0$, where a and c are real constants, has roots α and α^*.

a Given that $\text{Re}(\alpha) = 2$, find the value of a.

b Given that $\text{Im}(\alpha) = 3i$, find the value of c.

13 The equation $4x^2 + px + q = 0$, where p and q are real constants, has roots α and α^*.

a Given that $\text{Re}(\alpha) = -3$, find the value of p.

b Given that $\text{Im}(\alpha) \neq 0$, find the range of possible values of q.

Exercise B

1 α, β and γ are the roots of the cubic equation $2x^3 + 5x^2 - 2x + 3 = 0$. Find the values of:

a $\alpha + \beta + \gamma$ **b** $\alpha\beta\gamma$ **c** $\alpha\beta + \beta\gamma + \gamma\alpha$ **d** $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

2 α, β and γ are the roots of the cubic equation $x^3 + 5x^2 + 17x + 13 = 0$. Find the values of:

a $\alpha + \beta + \gamma$ **b** $\alpha\beta\gamma$ **c** $\alpha\beta + \beta\gamma + \gamma\alpha$ **d** $\alpha^2\beta^2\gamma^2$

3 α, β and γ are the roots of the cubic equation $7x^3 - 4x^2 - x + 6 = 0$. Find the values of:

a $\alpha + \beta + \gamma$ **b** $\alpha\beta\gamma$ **c** $\alpha^3\beta^3\gamma^3$ **d** $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

4 The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = \frac{3}{2}$, $\beta = \frac{1}{2}$ and $\gamma = 1$.
Find integer values for a, b, c and d.

5 The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = 1 + 3i$, $\beta = 1 - 3i$ and $\gamma = \frac{1}{2}$
Find integer values for a, b, c and d.

6 The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = \frac{5}{4}$, $\beta = -\frac{3}{2}$ and $\gamma = \frac{1}{2}$
Find integer values for a, b, c and d.

7 The cubic equation $16x^3 - kx^2 + 1 = 0$ has roots α, β and γ.

a Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(2 marks)**

b i Given that $\alpha = \beta$, find the roots of the equation. **(5 marks)**

ii Find the value of k. **(1 mark)**

8 The cubic equation $2x^3 - kx^2 + 30x - 13 = 0$ has roots α, β and γ.

a Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$, and express k in terms of α, β and γ. **(3 marks)**

b Given that $\alpha = 2 - 3i$, find the value of k. **(4 marks)**

9 The cubic equation $x^3 - mx + n = 0$ has roots 1, -4 and α.

a State, with a reason, whether α is real. **(1 mark)**

b Find the values of m, n and α. **(4 marks)**

10 The cubic equation $2x^3 - 10x^2 + 8x - k = 0$ has a root at $x = 3 - i$.

a Find the other two roots of the equation. **(4 marks)**

b Hence find the value of k. **(2 marks)**

11 The cubic equation $x^3 - 14x^2 + 56x - 64 = 0$ has roots α, $k\alpha$ and $k^2\alpha$ for some real constant k.
Find the values of α and k. **(5 marks)**

12 Given that the roots of $8x^3 + 12x^2 - cx + d = 0$ are α, $\frac{\alpha}{2}$ and $\alpha - 4$, find α, c and d. **(5 marks)**

13 Given that the roots of the cubic equation $2x^3 + 48x^2 + cx + d = 0$ are α, 2α and 3α, find the values of α, c and d. **(5 marks)**

Exercise C

1 α, β, γ and δ are the roots of the quartic equation $4x^4 + 3x^3 + 2x^2 - 5x - 4 = 0$. Without solving the equation, find the values of:

a $\alpha + \beta + \gamma + \delta$ **b** $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

c $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$ **d** $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

Hint $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta + \alpha\beta\gamma}{\alpha\beta\gamma\delta}$

2 α, β, γ and δ are the roots of the quartic equation $2x^4 + 4x^3 - 3x^2 - x + 2 = 0$. Find the values of:

a $\alpha + \beta + \gamma + \delta$ **b** $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ **c** $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

d $\alpha\beta\gamma\delta$ **e** $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

3 α, β, γ and δ are the roots of the quartic equation $x^4 + 3x^3 + 2x^2 - x + 4 = 0$. Find the values of:

a $\alpha + \beta + \gamma + \delta$ **b** $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ **c** $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

d $\alpha\beta\gamma\delta$ **e** $\alpha^2\beta^2\gamma^2\delta^2$

4 α, β, γ and δ are the roots of the quartic equation $7x^4 + 6x^3 - 5x^2 + 4x + 3 = 0$. Find the values of:

a $\alpha + \beta + \gamma + \delta$ **b** $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ **c** $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

d $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **e** $\alpha^3\beta^3\gamma^3\delta^3$

5 The roots of a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ are $\alpha = -\frac{3}{2}$, $\beta = -\frac{1}{2}$, $\gamma = -2$ and $\delta = \frac{2}{3}$. Find integer values for a, b, c, d and e.

6 The roots of a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ are $\alpha = -\frac{1}{2}$, $\beta = \frac{1}{3}$, $\gamma = 1 + \mathrm{i}$ and $\delta = 1 - \mathrm{i}$. Find integer values for a, b, c, d and e.

7 The roots of a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ are such that $\Sigma\alpha = \frac{17}{12}$, $\Sigma\alpha\beta = -\frac{25}{72}$, $\Sigma\alpha\beta\gamma = -\frac{53}{72}$ and $\alpha\beta\gamma\delta = -\frac{1}{6}$. Find integer values for a, b, c, d and e.

8 The quartic equation $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$ has roots α, $\alpha + k$, $\alpha + 2k$ and $\alpha + 3k$ for some real constant k. Solve the equation. **(7 marks)**

9 The quartic equation $3072x^4 - 2880x^3 + 840x^2 - 90x + 3 = 0$ has roots α, $r\alpha$, $r^2\alpha$ and $r^3\alpha$ for some real constant r. Solve the equation. **(7 marks)**

10 Three of the roots of the quartic equation $40x^4 + 90x^3 - 115x^2 + mx + n = 0$ are 1, -3 and $\frac{1}{2}$

a Find the fourth root. **(2 marks)**

b Find the values of m and n. **(4 marks)**

11 The quartic equation $2x^4 - 34x^3 + 202x^2 + dx + e = 0$ has roots α, $\alpha + 1$, $2\alpha + 1$ and $3\alpha + 1$.

a Find α. **(2 marks)**

b Find the values of d and e. **(4 marks)**

12 The equation $4x^4 - 19x^3 + px^2 + qx + 10 = 0$, $x \in \mathbb{C}$, $p, q \in \mathbb{R}$, has roots α, β, γ and δ. Given that $\gamma = 3 + \mathrm{i}$ and $\delta = \gamma^*$,

a show that $4\alpha + 4\beta + 5 = 0$ and that $4\alpha\beta - 1 = 0$. **(2 marks)**

b Hence find all the roots of the quartic equation and find the values of p and q. **(5 marks)**

c Show these roots on an Argand diagram. **(3 marks)**

13 A quartic equation $6x^4 - 10x^3 + 3x^2 + 6x - 40 = 0$ has roots α, β, γ and δ.

a Show that $\frac{1-3i}{2}$ is one root of the equation. **(3 marks)**

b Without solving the equation, find the other roots. **(5 marks)**

c Show these roots on an Argand diagram. **(3 marks)**

Exercise D

1 A quadratic equation has roots α and β. Given that $\alpha + \beta = 4$ and $\alpha\beta = 3$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta}$ **b** $\alpha^2\beta^2$ **c** $\alpha^2 + \beta^2$ **d** $\alpha^3 + \beta^3$

2 A quadratic equation has roots α and β. Given that $\alpha + \beta = -\frac{2}{3}$ and $\alpha\beta = \frac{3}{4}$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta}$ **b** $\alpha^2\beta^2$ **c** $\alpha^2 + \beta^2$ **d** $\alpha^3 + \beta^3$

3 A quadratic equation has roots α and β. Given that $\alpha + \beta = \frac{5}{4}$ and $\alpha\beta = -\frac{1}{3}$, find:

a $(\alpha + 2)(\beta + 2)$ **b** $(\alpha - 4)(\beta - 4)$ **c** $(\alpha^2 + 1)(\beta^2 + 1)$

4 A cubic equation has roots α, β and γ. Given that $\alpha + \beta + \gamma = 2$, $\alpha\beta + \beta\gamma + \gamma\alpha = -3$ and $\alpha\beta\gamma = 4$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **b** $\alpha^2 + \beta^2 + \gamma^2$ **c** $\alpha^3 + \beta^3 + \gamma^3$ **d** $(\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2$

5 A cubic equation has roots α, β and γ. Given that $\Sigma\alpha = \frac{3}{2}$, $\Sigma\alpha\beta = -\frac{4}{3}$ and $\alpha\beta\gamma = \frac{1}{2}$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **b** $\alpha^2 + \beta^2 + \gamma^2$ **c** $\alpha^3 + \beta^3 + \gamma^3$ **d** $\alpha^3\beta^3\gamma^3$

6 A cubic equation has roots α, β and γ. Given that $\alpha + \beta + \gamma = -\frac{1}{2}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{3}{4}$ and $\alpha\beta\gamma = -\frac{2}{5}$, find:

a $(\alpha + 2)(\beta + 2)(\gamma + 2)$ **b** $(\alpha - 3)(\beta - 3)(\gamma - 3)$ **c** $(1 - \alpha)(1 - \beta)(1 - \gamma)$

d $(\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2$ **e** $(\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3$

7 A quartic equation has roots α, β, γ and δ. Given that $\alpha + \beta + \gamma + \delta = 3$, $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = 5$, $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -4$ and $\alpha\beta\gamma\delta = -2$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **b** $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ **c** $\alpha^4\beta^4\gamma^4\delta^4$

8 A quartic equation has roots α, β, γ and δ. Given that $\Sigma\alpha = \frac{1}{2}$, $\Sigma\alpha\beta = -\frac{3}{4}$, $\Sigma\alpha\beta\gamma = -\frac{1}{5}$ and $\alpha\beta\gamma\delta = \frac{4}{3}$, find:

a $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **b** $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ **c** $\alpha^3\beta^3\gamma^3\delta^3$

d $(\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2 + (\gamma\delta)^2 + (\alpha\delta)^2 + (\beta\delta)^2$

e $(\alpha\beta\gamma)^2 + (\alpha\beta\delta)^2 + (\alpha\gamma\delta)^2 + (\beta\gamma\delta)^2$

9 A quartic equation has roots α, β, γ and δ. Given that $\Sigma\alpha = -\frac{1}{2}$, $\Sigma\alpha\beta = -\frac{1}{3}$, $\Sigma\alpha\beta\gamma = \frac{1}{4}$ and $\alpha\beta\gamma\delta = \frac{3}{2}$, find:

a $(\alpha + 1)(\beta + 1)(\gamma + 1)(\delta + 1)$ **b** $(2 - \alpha)(2 - \beta)(2 - \gamma)(2 - \delta)$

10 The roots of the equation $x^3 - 6x^2 + 9x - 15 = 0$ are α, β and γ.

a Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(1 mark)**

b Hence find the values of:

i $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **(2 marks)**

ii $\alpha^2 + \beta^2 + \gamma^2$ **(2 marks)**

iii $(\alpha - 1)(\beta - 1)(\gamma - 1)$ **(3 marks)**

11 The roots of the equation $2x^3 + 4x^2 + 7 = 0$ are α, β and γ.

a Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(1 mark)**

b Hence find the values of:

i $\alpha^2 + \beta^2 + \gamma^2$ **(2 marks)**

ii $\alpha^3\beta^3\gamma^3$ **(2 marks)**

iii $(\alpha + 2)(\beta + 2)(\gamma + 2)$ **(3 marks)**

12 Show that $\alpha^3 + \beta^3 + \gamma^3 \equiv (\alpha + \beta + \gamma)^3 - 3(\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha) + 3\alpha\beta\gamma$.

13 The roots of the equation $3x^3 - px + 11 = 0$ are α, β and γ.

a Given that $\alpha\beta + \beta\gamma + \gamma\alpha = 4$, write down the value of p. **(1 mark)**

b Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$. **(1 mark)**

c Hence find the value of $(3 - \alpha)(3 - \beta)(3 - \gamma)$. **(3 marks)**

14 The roots of the equation $x^4 + 2x^2 - x + 3 = 0$ are α, β, γ and δ.

a Write down the values of $\Sigma\alpha$, $\Sigma\alpha\beta$, $\Sigma\alpha\beta\gamma$ and $\alpha\beta\gamma\delta$. **(1 mark)**

b Hence find the values of:

i $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **(3 marks)**

ii $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ **(3 marks)**

iii $(\alpha + 1)(\beta + 1)(\gamma + 1)(\delta + 1)$ **(3 marks)**

15 The roots of the equation $ax^4 + 3x^3 + 2x^2 + x - 6 = 0$ are α, β, γ and δ.

a Given that $\alpha\beta\gamma\delta = -3$, write down the value of a. **(1 mark)**

b Write down the values of $\Sigma\alpha$, $\Sigma\alpha\beta$ and $\Sigma\alpha\beta\gamma$. **(1 mark)**

c Hence find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$ **(3 marks)**

16 Prove that if a quartic equation has roots α, β, γ and δ then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 \equiv (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$.

Exercise E

1 The cubic equation $x^3 - 7x^2 + 6x + 5 = 0$ has roots α, β and γ.
Find equations with roots:

a $(\alpha + 1)$, $(\beta + 1)$ and $(\gamma + 1)$ **b** 2α, 2β and 2γ

2 The cubic equation $3x^3 - 4x^2 - 5x + 1 = 0$ has roots α, β and γ.
Find equations with roots:

a $(\alpha - 3)$, $(\beta - 3)$ and $(\gamma - 3)$ **b** $\frac{\alpha}{2}$, $\frac{\beta}{2}$ and $\frac{\gamma}{2}$

3 The cubic equation $x^3 - 3x^2 + 4x - 7 = 0$ has roots α, β and γ.
Without solving the equation, find the equation with roots $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$.
Give your answer in the form $aw^3 + bw^2 + cw + d = 0$ where a, b, c and d are integers to be determined. **(5 marks)**

4 The cubic equation $x^3 + 4x^2 - 4x + 2 = 0$ has roots α, β and γ.
Without solving the equation, find the equation with roots $(2\alpha - 1)$, $(2\beta - 1)$ and $(2\gamma - 1)$.
Give your answer in the form $w^3 + pw^2 + qw + r = 0$ where p, q and r are integers to be found. **(5 marks)**

5 The cubic equation $3x^3 - x^2 + 2x - 5 = 0$ has roots α, β and γ.
Without solving the equation, find the equation with roots $(3\alpha + 1)$, $(3\beta + 1)$ and $(3\gamma + 1)$.
Give your answer in the form $aw^3 + bw^2 + cw + d = 0$ where a, b, c and d are integers to be determined. **(5 marks)**

6 The quartic equation $2x^4 + 4x^3 - 5x^2 + 2x - 1 = 0$ has roots α, β, γ and δ. Find equations with integer coefficients that have roots:

a 3α, 3β, 3γ and 3δ **b** $(\alpha - 1)$, $(\beta - 1)$, $(\gamma - 1)$ and $(\delta - 1)$

7 The quartic equation $x^4 + 2x^3 - 3x^2 + 4x + 5 = 0$ has roots α, β, γ and δ.
Without solving the equation, find equations with integer coefficients that have roots:

a 2α, 2β, 2γ and 2δ **(6 marks)**

b $(\alpha - 2)$, $(\beta - 2)$, $(\gamma - 2)$ and $(\delta - 2)$ **(6 marks)**

8 The quartic equation $3x^4 + 5x^3 - 4x^2 - 3x + 1 = 0$ has roots α, β, γ and δ.
Without solving the equation, find equations with integer coefficients that have roots:

a 3α, 3β, 3γ and 3δ **(6 marks)**

b $(\alpha + 1)$, $(\beta + 1)$, $(\gamma + 1)$ and $(\delta + 1)$ **(6 marks)**

Exercise F

1 The roots of a quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ are $\alpha = \frac{1}{5}$, $\beta = -\frac{2}{5}$, $\gamma = -\frac{3}{5}$ and $\delta = -\frac{1}{2}$
Find integer values for a, b, c, d and e.

2 The cubic equation $x^3 + px^2 + 37x - 52 = 0$ has roots α, β and γ.

a Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$, and express p in terms of α, β and γ. **(3 marks)**

b Given that $\alpha = 3 - 2i$, find the value of p. **(4 marks)**

3 The cubic equation $2x^3 + 5x^2 - 2x + q = 0$ has a root at $x = -2 + i$.

a Find the other two roots of the equation. **(4 marks)**

b Hence find the value of q. **(2 marks)**

4 The quartic equation $x^4 - 40x^3 + 510x^2 - 2200x + 1729 = 0$ has roots α, $\alpha + 2k$, $\alpha + 4k$ and $\alpha + 6k$ for some real constant k. Solve the equation. **(7 marks)**

5 Three of the roots of the quartic equation $24x^4 - 58x^3 + 17x^2 + dx + e = 0$ are $\frac{1}{2}$, $-\frac{1}{3}$ and 2.

a Find the fourth root. **(2 marks)**

b Find the values of d and e. **(4 marks)**

6 The equation $x^4 + 2x^3 + mx^2 + nx + 85 = 0$, $x \in \mathbb{C}$, $m, n \in \mathbb{R}$, has roots α, β, γ and δ.
Given that $\alpha = -2 + i$ and $\beta = \alpha^*$,

a show that $\gamma + \delta - 2 = 0$ and that $\gamma\delta - 17 = 0$. **(2 marks)**

b Hence find all the roots of the quartic equation and find the values of m and n. **(5 marks)**

c Show these roots on an Argand diagram. **(3 marks)**

7 A quartic equation $4x^4 - 16x^3 + 115x^2 + 4x - 29 = 0$ has roots α, β, γ and δ.

a Show that $2 - 5i$ is one root of the equation. **(3 marks)**

b Without solving the equation, find the other roots. **(5 marks)**

c Show these roots on an Argand diagram. **(3 marks)**

8 The roots of the equation $2x^3 - 5x^2 + 11x - 9 = 0$ are α, β and γ.

a Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **(1 mark)**

b Hence find the values of:

i $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **(2 marks)**

ii $\alpha^2 + \beta^2 + \gamma^2$ **(2 marks)**

iii $(\alpha - 1)(\beta - 1)(\gamma - 1)$ **(3 marks)**

9 The roots of the equation $px^4 + 12x^3 + 6x^2 + 5x - 7 = 0$ are α, β, γ and δ.

a Given that $\alpha\beta\gamma\delta = -1$, write down the value of p. **(1 mark)**

b Write down the values of $\Sigma\alpha$, $\Sigma\alpha\beta$ and $\Sigma\alpha\beta\gamma$. **(1 mark)**

c Hence find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. **(3 marks)**

10 The roots of the equation $5x^3 + cx + 21 = 0$ are α, β and γ.

a Given that $\alpha\beta + \beta\gamma + \gamma\alpha = -6$, write down the value of c. **(1 mark)**

b Write down values for $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$. **(1 mark)**

c Hence find the value of $(1 - \alpha)(1 - \beta)(1 - \gamma)$. **(3 marks)**

11 The cubic equation $2x^3 + 5x^2 + 7x - 2 = 0$ has roots α, β and γ.
Without solving the equation, find the equation with roots $(3\alpha + 1)$, $(3\beta + 1)$ and $(3\gamma + 1)$.
Give your answer in the form $pw^3 + qw^2 + rw + s = 0$ where p, q, r and s are integers to be found. **(5 marks)**

12 The quartic equation $6x^4 - 2x^3 - 5x^2 + 7x + 8 = 0$ has roots α, β, γ and δ.
Without solving the equation, find equations with integer coefficients that have roots:

a 2α, 2β, 2γ and 2δ **(6 marks)**

b $(3\alpha - 2)$, $(3\beta - 2)$, $(3\gamma - 2)$ and $(3\delta - 2)$ **(6 marks)**