Differentiation - 4

1.

(i) Find the coordinates of the stationary point on the curve $y = 3x^2 - \frac{6}{x} - 2$. **[5]**

(ii) Determine whether the stationary point is a maximum point or a minimum point. **[2]**

2.

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\frac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

3.

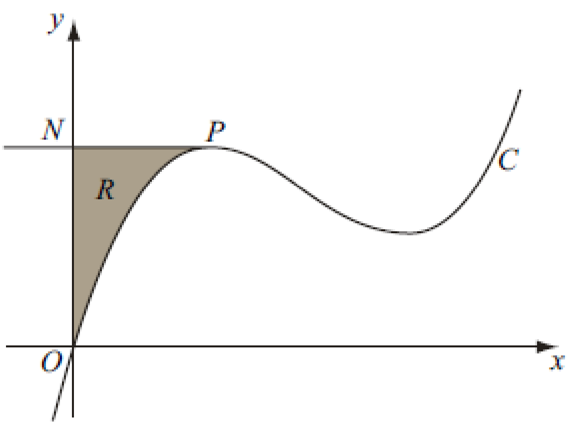

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

4.

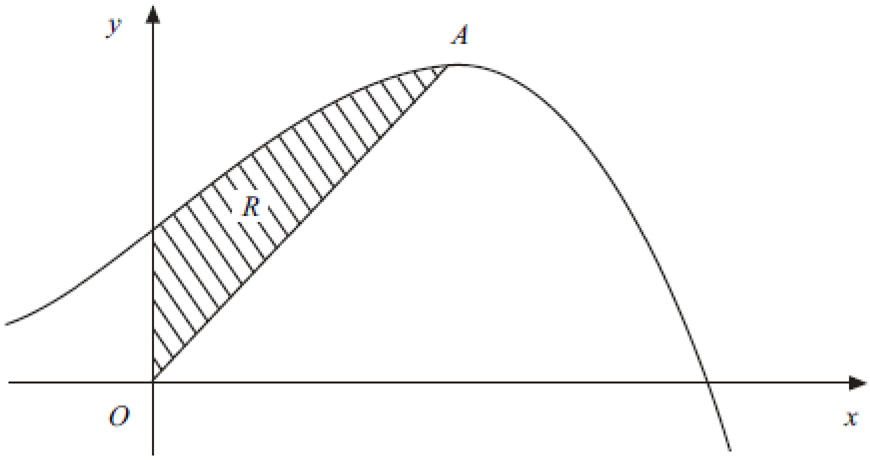

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

5.

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)