Linear Transformations

Imporatnt Points:

1.

You can define a transformation in two dimensions by describing how a general point with position vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is transformed. The new point is called the **image**.

2.

- **Linear transformations always map the origin onto itself.**
- **Any linear transformation can be represented by a matrix.**
- **The linear transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ can be represented by the matrix $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ since $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$.**

Note You can transform any point P by multiplying the transformation matrix by the position vector of P.

3.

Any linear transformation can be defined by the effect it has on unit vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

The transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ will map $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} a \\ c \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} b \\ d \end{pmatrix}$.

You can visualise this transformation by considering the unit square:

4.

Points which are mapped onto themselves under the given transformation are called **invariant points**. Lines which map onto themselves are called **invariant lines**.

5.

In reflections, each point on the mirror line is an invariant point.

The mirror line and any lines that are perpendicular the mirror line are invariant lines.

6.

- **The matrix representing a rotation through angle θ anticlockwise about the origin is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$**

Note This general rotation matrix is given in the formulae booklet.

The only invariant point is the origin (0, 0). For $\theta \neq 180°$, there are no invariant lines. For $\theta = 180°$, any line passing through the origin is an invariant line.