Full Name: …….. **Date: 01 February 2026**

Test

1.

$$z = 2 - i\sqrt{3}$$

Use algebra to express

(a) $z + z^2$ in the form $a + bi\sqrt{3}$, where a and b are integers,

(3)

(b) $\frac{z+7}{z-1}$ in the form $c + di\sqrt{3}$, where c and d are integers.

(4)

2.

(a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} \left(r^3 + 6r - 3\right) = \frac{1}{4} n^2 (n^2 + 2n + 13)$$

for all positive integers n.

(5)

(b) Hence find the exact value of

$$\sum_{r=16}^{30} \left(r^3 + 6r - 3\right)$$

(2)

3.

Use an algebraic method to find the square roots of $11 + (12\sqrt{5})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

- End of Test -