Series

Exercise A

1 Evaluate:

a $\sum_{r=0}^{3}(2r+1)$ **b** $\sum_{r=1}^{40} r$ **c** $\sum_{r=1}^{20} r$ **d** $\sum_{r=1}^{99} r$

e $\sum_{r=10}^{40} r$ **f** $\sum_{r=100}^{200} r$ **g** $\sum_{r=21}^{40} r$ **h** $\sum_{r=1}^{k} r + \sum_{r=k+1}^{80} r,\ 0 < k < 80$

2 Given that $\sum_{r=1}^{n} r = 528$, find the value of n. **(4 marks)**

3 Given that $\sum_{r=1}^{k} r = \frac{1}{2}\sum_{r=1}^{20} r$, find the value of k. **(4 marks)**

4 **a** Find an expression for $\sum_{r=1}^{2n-1} r$. **(3 marks)**

b Hence show that $\sum_{r=n+1}^{2n-1} r = \frac{3}{2}n(n-1),\ n \geqslant 2$. **(3 marks)**

5 Show that $\sum_{r=n-1}^{2n} r = \frac{1}{2}(n+2)(3n-1),\ n \geqslant 1$. **(5 marks)**

6 **a** Show that $\sum_{r=1}^{n^2} r - \sum_{r=1}^{n} r = \frac{1}{2}n(n^3-1)$.

b Hence evaluate $\sum_{r=10}^{81} r$.

Hint Use your result from part **a**.

7 Calculate the sum of each series:

a $\sum_{r=1}^{55}(3r-1)$ **b** $\sum_{r=1}^{90}(2-7r)$ **c** $\sum_{r=1}^{46}(9+2r)$

8 Show that:

a $\sum_{r=1}^{n}(3r+2) = \frac{1}{2}n(3n+7)$ **b** $\sum_{r=1}^{2n}(5r-4) = n(10n-3)$

c $\sum_{r=1}^{n+2}(2r+3) = (n+2)(n+6)$ **d** $\sum_{r=3}^{n}(4r+5) = (2n+11)(n-2)$

9 **a** Show that $\sum_{r=1}^{k}(4r-5) = 2k^2 - 3k$. **(5 marks)**

b Find the smallest value of k for which $\sum_{r=1}^{k}(4r-5) > 4850$. **(4 marks)**

10 Given that $\mathrm{f}(r) = ar + b$ and $\sum_{r=1}^{n}\mathrm{f}(r) = \frac{1}{2}n(7n+1)$, find the constants a and b. **(5 marks)**

11 **a** Show that $\sum_{r=1}^{4n-1}(3r+1) = 24n^2 - 2n - 1,\ n \geqslant 1$. **(5 marks)**

b Hence calculate $\sum_{r=1}^{99}(3r+1)$. **(2 marks)**

12 **a** Show that $\sum_{r=1}^{2k+1}(4-5r) = -(2k+1)(5k+1),\ k \geqslant 0$. **(5 marks)**

b Hence evaluate $\sum_{r=1}^{25}(4-5r)$. **(2 marks)**

c Find the value of $\sum_{r=1}^{15}(5r-4)$. **(1 mark)**

13 Given that $\sum_{r=1}^{n} \mathrm{f}(r) = n^2 + 4n$, deduce an expression for $\mathrm{f}(r)$ in terms of r.

14 $\mathrm{f}(r) = qr + b$, where a and b are rational constants.

Given that $\sum_{r=1}^{4} \mathrm{f}(r) = 36$ and $\sum_{r=1}^{6} \mathrm{f}(r) = 78$,

a find an expression for $\sum_{r=1}^{n} \mathrm{f}(r)$ **(6 marks)**

b hence calculate $\sum_{r=1}^{10} \mathrm{f}(r)$. **(2 marks)**

Exercise B

1 Evaluate:

a $\sum_{r=1}^{4} r^2$ **b** $\sum_{r=1}^{40} r^2$ **c** $\sum_{r=21}^{40} r^2$ **d** $\sum_{r=1}^{99} r^3$

e $\sum_{r=1}^{100} r^3$ **f** $\sum_{r=100}^{200} r^3$ **g** $\sum_{r=1}^{k} r^2 + \sum_{r=k+1}^{80} r^2,\ 0 < k < 80.$

2 Show that:

a $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$ **b** $\sum_{r=1}^{2n-1} r^2 = \frac{1}{3}n(2n-1)(4n-1)$

c $\sum_{r=n}^{2n} r^2 = \frac{1}{6}n(n+1)(14n+1)$

3 Show that, for any $k \in \mathbb{N}$, $\sum_{r=1}^{n+k} r^3 = \frac{1}{4}(n+k)^2(n+k+1)^2$

4 a Show that $\sum_{r=n+1}^{3n} r^3 = n^2(4n+1)(5n+2)$ **(3 marks)**

b Hence evaluate $\sum_{r=11}^{30} r^3$. **(2 marks)**

5 a Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. **(3 marks)**

b Hence evaluate $\sum_{r=30}^{60} r^3$. **(2 marks)**

6 Evaluate:

a $\sum_{m=1}^{30} (m^2 - 1)$ **b** $\sum_{r=1}^{40} r(r+4)$ **c** $\sum_{r=1}^{80} r(r^2+3)$ **d** $\sum_{r=11}^{35} (r^3 - 2)$

7 a Show that $\sum_{r=1}^{n} (r+2)(r+5) = \frac{1}{3}n(n^2 + 12n + 41)$ **(4 marks)**

b Hence calculate $\sum_{r=10}^{50} (r+2)(r+5)$. **(3 marks)**

8 a Show that $\sum_{r=1}^{n} (r^2 + 3r + 1) = \frac{1}{3}n(n+a)(n+b)$, where a and b are integers to be found. **(4 marks)**

b Hence evaluate $\sum_{r=19}^{40} (r^2 + 3r + 1)$. **(3 marks)**

9 **a** Show that $\sum_{r=1}^{n} r^2(r-1) = \frac{1}{12}n(n+1)(3n^2-n-2)$. **(4 marks)**

b Hence show that $\sum_{r=1}^{2n-1} r^2(r-1) = \frac{1}{3}n(2n-1)(6n^2-7n+1)$ **(4 marks)**

10 **a** Show that $\sum_{r=1}^{n} (r+1)(r+3) = \frac{1}{6}n(2n^2+an+b)$, where a and b are integers to be found. **(4 marks)**

b Hence find an expression, only in terms of n, for $\sum_{r=n+1}^{2n} (r+1)(r+3)$. **(3 marks)**

11 **a** Show that $\sum_{r=1}^{n} (r+3)(r+4) = \frac{1}{3}n(n^2+an+b)$, where a and b are integers to be found. **(4 marks)**

b Hence find an expression, only in terms of n, for $\sum_{r=n+1}^{3n} (r+3)(r+4)$. **(3 marks)**

12 **a** Show that $\sum_{r=1}^{n} r(r+3)^2 = \frac{1}{4}n(n+1)(n^2+an+b)$, where a and b are integers to be found. **(5 marks)**

b Hence evaluate $\sum_{r=10}^{20} r(r+3)^2$. **(3 marks)**

13 **a** Show that, for any $k \in \mathbb{N}$, $\sum_{r=1}^{kn}(2r-1) = k^2n^2$.

b Hence find a value of n such that $\sum_{r=1}^{5n}(2r-1) = \sum_{r=1}^{n} r^3$.

14 **a** Show that $\sum_{r=1}^{n}(r^3-r^2) = \frac{1}{12}n(n+1)(n-1)(3n+2)$. **(4 marks)**

b Hence find the value of n that satisfies $\sum_{r=1}^{n}(r^3-r^2) = \sum_{r=1}^{n} 7r$. **(5 marks)**

Exercise C

Throughout this exercise you may assume the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$.

1 Evaluate:

a $\sum_{r=1}^{10} r$ **b** $\sum_{r=10}^{50} r$ **c** $\sum_{r=1}^{10} r^2$ **d** $\sum_{r=1}^{10} r^3$

e $\sum_{r=26}^{50} r^2$ **f** $\sum_{r=50}^{100} r^3$ **g** $\sum_{r=1}^{60} r + \sum_{r=1}^{60} r^2$

2 Write each of the following as an expression in terms of n.

a $\sum_{r=1}^{n}(3r-5)$ **b** $\sum_{r=1}^{n}(r^2+r)$ **c** $\sum_{r=1}^{n}(3r^2+7r)$

d $\sum_{r=1}^{n}(4r^3+6r^2)$ **e** $\sum_{r=1}^{n}(r^2-2r)$ **f** $\sum_{r=1}^{n}(r^2-3r)$

g $\sum_{r=1}^{n}(r^2-5)$ **h** $\sum_{r=1}^{n}(2r^3+3r^2+r+4)$

3 Evaluate $\sum_{r=1}^{30} r(3r-1)$. **(5 marks)**

4 **a** Show that $\sum_{r=1}^{n} r^2(r-3) = \frac{1}{4}n(n+1)(n^2+an+b)$, where a and b are integers to be found. **(4 marks)**

b Hence evaluate $\sum_{r=1}^{20} r^2(r-3)$. **(2 marks)**

5 **a** Show that $\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(an+b)(an-b)$, where a and b are integers to be found. **(5 marks)**

b Hence find $\sum_{r=1}^{2n} (2r-1)^2$. **(2 marks)**

6 **a** Show that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(an+b)$, where a and b are integers to be found. **(4 marks)**

b Hence evaluate $\sum_{r=15}^{30} r(r+2)$. **(3 marks)**

7 **a** Show that $\sum_{r=n+1}^{2n} r^2 = \frac{1}{6}n(2n+1)(an+b)$, where a and b are integers to be found. **(4 marks)**

b Hence evaluate $\sum_{r=16}^{30} r^2$. **(2 marks)**

8 **a** Show that $\sum_{r=1}^{n} (r^2-r-1) = \frac{1}{3}n(n^2-4)$. **(4 marks)**

b Hence evaluate $\sum_{r=10}^{40} (r^2-r-1)$. **(3 marks)**

c Find the value of n such that $\sum_{r=1}^{n} (r^2-r-1) = \sum_{r=1}^{2n} r$. **(5 marks)**

9 **a** Show that $\sum_{r=1}^{n} r(2r^2+1) = \frac{1}{2}n(n+1)(n^2+n+1)$. **(4 marks)**

b Hence show that there are no values of n that satisfy $\sum_{r=1}^{n} r(2r^2+1) = \sum_{r=1}^{n} (100r^2-r)$. **(6 marks)**

10 **a** Show that $\sum_{r=1}^{n} r(r+1)^2 = \frac{1}{12}n(n+1)(n+2)(an+b)$, where a and b are integers to be found. **(5 marks)**

b Hence find the value of n that satisfies $\sum_{r=1}^{n} r(r+1)^2 = \sum_{r=1}^{n} 70r$. **(6 marks)**

11 Find the value of n that satisfies $\sum_{r=1}^{n} r^2 = \sum_{r=1}^{n+1} (9r+1)$. **(7 marks)**