Probability

Exercise A

1. Which of these events are mutually exclusive? For those events that are mutually exclusive, find P(A or B).

   (a) On a fair six-sided dice:

   (i) Event A: rolling a multiple of 3; Event B: rolling a multiple of 4

   (ii) Event A: rolling an even number; Event B: rolling a multiple of 5

   (b) One card is selected from a standard pack of 52 cards.

   (i) Event A: selecting a king; Event B: selecting a red card

   (ii) Event A: selecting an ace; Event B: selecting a spade

   (c) Two fair dice are rolled and the scores added.

   (i) Event A: the total is a multiple of 6; Event B: the total is less than 5.

   (ii) Event A: the total is greater than 7; Event B: the total is less than 9.

2. Which pairs of events from question 1 are independent? For those that are, calculate P(A and B).

3. Daniel has three blocks with letters C, A and T written on them. He arranges the blocks in a row randomly.

   (a) Write down all possible arrangements of the three letters.

   (b) Find the probability that the blocks make the word ‘CAT’ or ‘ACT’.

4. A fair six-sided dice is rolled once. Define events

   A: the dice shows an even number; B: the dice shows a prime number.

   (a) Find P(A and B).

   (b) Determine whether events A and B are independent.

5. 300 students in years 9, 10 and 11 at a school were asked to say which of Biology, Chemistry and Physics is their favourite science. The results are shown in this table.

Year group	Biology	Chemistry	Physics	Total
Year 9	41	29	27	97
Year 10	35	36	34	105
Year 11	37	30	31	98
Total	113	95	92	300

   (a) Find the probability that a randomly chosen student

(i) prefers Chemistry (ii) is in Year 11 and doesn't prefer Biology

(b) Determine whether the event 'the student is in year 9' and the event 'the student's favourite science is Physics' are independent.

6. A four-sided spinner, with numbers 1 to 4 written on it, is spun three times. Find the probability of getting either three 1s or three 4s.

Exercise B

1.

A and B are two events and $P(A) = 0.5$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$.

Find

a $P(A \cup B)$, **b** $P(B')$,

c $P(A \cap B')$, **d** $P(A \cup B')$.

2.

A and C are two events and $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$.

Find

a $P(A \cap B)$, **b** $P(A')$,

c $P(A \cup B')$, **d** $P(A' \cup B)$.

3.

If A and B are two events and $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cup B) = 0.8$, find:

(a) $P(A \cap B)$ (b) $P(A' \cap B)$ (c) $P(A \cap B')$

(d) $P(A' \cap B')$ (e) $P(A \cup B')$ (f) $P(A' \cup B)$.

4.

If S and T are two events and $P(T) = 0.4$, $P(S \cap T) = 0.15$ and $P(S' \cap T') = 0.5$, find:

(a) $P(S \cap T')$ (b) $P(S)$ (c) $P(S \cup T)$

(d) $P(S' \cap T)$ (e) $P(S' \cup T')$.

5.

C and D are two events and $P(D) = 0.4$, $P(C \cap D) = 0.15$ and $P(C' \cap D') = 0.1$.

Find

a $P(C' \cap D)$, **b** $P(C \cap D')$,

c $P(C)$, **d** $P(C' \cap D')$.

6.

There are two events T and Q where $P(T) = P(Q) = 3P(T \cap Q)$ and $P(T \cup Q) = 0.75$.

Find

a $P(T \cap Q)$, **b** $P(T)$, **c** $P(Q')$,

d $P(T' \cap Q')$, **e** $P(T \cap Q')$.

7.

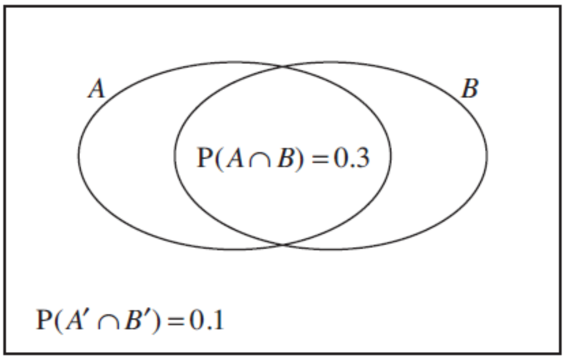
The events M and N are such that $P(M) = P(N) = 2P(M \cap N)$.
Given that $P(M \cup N) = 0.6$, find:

(a) $P(M \cap N)$ (b) $P(M)$ (c) $P(M' \cap N')$

(d) $P(M \cap N')$.

8.

The Venn diagram illustrates the occurrence of two events A and B.



You are given that $P(A \cap B) = 0.3$ and that the probability that neither A nor B occurs is 0.1. You are also given that $P(A) = 2P(B)$.

Find $P(B)$. [3]

9.

A survey of all the households in the town of Bury was carried out. The survey showed that 70% have a freezer and 20% have a dishwasher and 80% have either a dishwasher or a freezer or both appliances. Find the probability that a randomly chosen household in Bury has both appliances.

10.

The probability that a child in a school has blue eyes is 0.27 and the probability they have blonde hair is 0.35. The probability that the child will have blonde hair or blue eyes or both is 0.45. A child is chosen at random from the school. Find the probability that the child has

a blonde hair and blue eyes,

b blonde hair but not blue eyes,

c neither feature.

11.

A patient going in to a doctor's waiting room reads *Hiya* Magazine with probability 0.6 and *Dakor* Magazine with probability 0.4. The probability that the patient reads either one or both of the magazines is 0.7. Find the probability that the patient reads

a both magazines, **b** *Hiya* Magazine only.

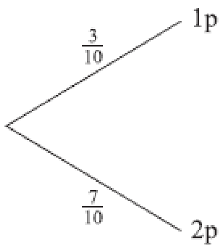
Exercise C

1.

A bag contains three 1p coins and seven 2p coins. Coins are removed at random one at a time, **without** replacement, until the total value of the coins removed is **at least** 3p. Then no more coins are removed.

(i) Copy and complete the probability tree diagram. [5]

First coin



Find the probability that

(ii) exactly two coins are removed, [3]

(iii) the total value of the coins removed is 4p. [3]

2.

(i) A bag contains 12 red discs and 10 black discs. Two discs are removed at random, without replacement. Find the probability that both discs are red. [2]

(ii) Another bag contains 7 green discs and 8 blue discs. Three discs are removed at random, without replacement. Find the probability that exactly two of these discs are green. [3]

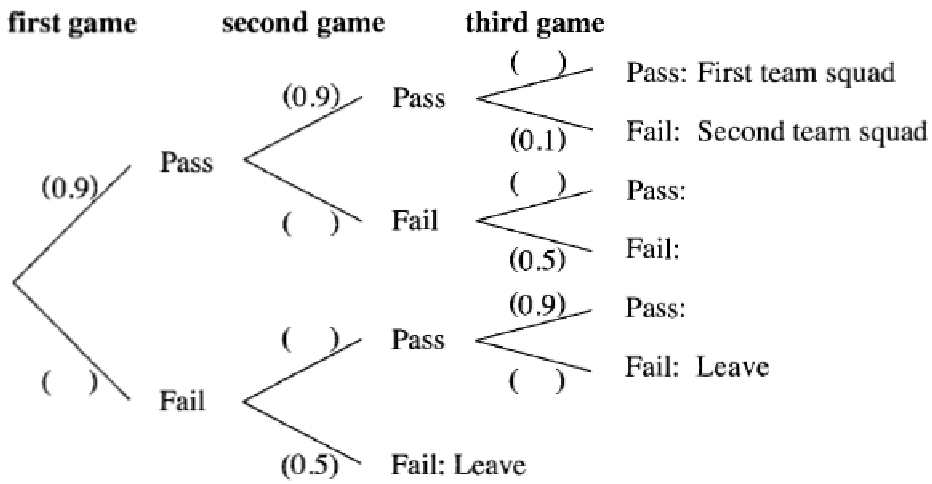
(iii) A third bag contains 45 discs, each of which is either yellow or brown. Two discs are removed at random, without replacement. The probability that both discs are yellow is $\frac{1}{15}$. Find the number of yellow discs which were in the bag at first. [4]

3.

Mancaster Hockey Club invite prospective new players to take part in a series of three trial games. At the end of each game the performance of each player is assessed as pass or fail. Players who achieve a pass in all three games are invited to join the first team squad. Players who achieve a pass in two games are invited to join the second team squad. Players who fail in two games are asked to leave. This may happen after two games.

- The probability of passing the first game is 0.9
- Players who pass any game have probability 0.9 of passing the next game
- Players who fail any game have probability 0.5 of failing the next game

(i) On the insert, complete the tree diagram which illustrates the information above. [2]



(ii) Find the probability that a randomly selected player

(*A*) is invited to join the first team squad, [2]

(*B*) is invited to join the second team squad. [3]

(iii) Hence write down the probability that a randomly selected player is asked to leave. [1]

4.

A bag contains 5 black discs and 3 red discs. A disc is selected at random from the bag. If it is red it is replaced in the bag. If it is black, it is not replaced. A second disc is now selected at random from the bag.

Find the probability that

(i) the second disc is black, given that the first disc was black, [1]

(ii) the second disc is black, [3]

(iii) the two discs are of different colours. [3]

5.

A bag contains 6 white discs and 4 blue discs. Discs are removed at random, one at a time, **without** replacement.

Find the probability that

(a) the second disc is blue, given that the first disc was blue, [1]

(b) the second disc is blue, [3]

(c) the third disc is blue, given that the first disc was blue. [3]

6.

Jenny and John are each allowed two attempts to pass an examination.

(i) Jenny estimates that her chances of success are as follows.

- The probability that she will pass on her first attempt is $\frac{2}{3}$.
- If she fails on her first attempt, the probability that she will pass on her second attempt is $\frac{3}{4}$.

Calculate the probability that Jenny will pass. [3]

(ii) John estimates that his chances of success are as follows.

- The probability that he will pass on his first attempt is $\frac{2}{3}$.
- Overall, the probability that he will pass is $\frac{5}{6}$.

Calculate the probability that if John fails on his first attempt, he will pass on his second attempt. [3]

7.

Jimmy and Alan are playing a tennis match against each other. The winner of the match is the first player to win three sets. Jimmy won the first set and Alan won the second set. For each of the remaining sets, the probability that Jimmy wins a set is

- 0.7 if he won the previous set,
- 0.4 if Alan won the previous set.

It is not possible to draw a set.

(i) Draw a probability tree diagram to illustrate the possible outcomes for each of the remaining sets. **[3]**

(ii) Find the probability that Alan wins the match. **[3]**

(iii) Find the probability that the match ends after exactly four sets have been played. **[2]**
