Complex Numbers 1 - Answers

Exercise A

1 **a** $3i$ **b** $7i$ **c** $11i$ **d** $100i$
e $15i$ **f** $i\sqrt{5}$ **g** $2i\sqrt{3}$ **h** $3i\sqrt{5}$
i $10i\sqrt{2}$ **j** $7i\sqrt{3}$

2 **a** $13 + 11i$ **b** $5 + 2i$ **c** $4 + i$ **d** $3 + 2i$
e $9 + 9i$ **f** $7 - 4i$ **g** $4 + 2i$ **h** $2\sqrt{2} + 2i$
i $11 - 5i$ **j** 0

3 **a** $14 + 4i$ **b** $24 - 12i$ **c** $12 + 5i$ **d** $24 + 7i$
e $3 - 2i$ **f** $3 + 5i$ **g** $3 + \frac{11}{3}i$ **h** $-\frac{11}{2} + \frac{7}{4}i$

4 **a** $2\sqrt{2} - i\sqrt{2}$ **b** $(-1 + \sqrt{3}) + (3 - 3\sqrt{3})i$

5 **a** $-12i$ **b** 14

6 $a = -10, b = 11$

7 **a** $-3 + 4i$ **b** $28 - 12i$ **c** $43 - 13i$

8 **a** $z + w = (a + bi) + (a - bi) = 2a$
b $z - w = (a + bi) - (a - bi) = 2bi$

Exercise B

1 **a** $z = \pm 11i$ **b** $z = \pm 2i\sqrt{10}$ **c** $z = \pm 2i\sqrt{15}$
d $z = \pm 2i\sqrt{7}$ **e** $z = \pm 2i\sqrt{6}$ **f** $z = \pm\frac{1}{2}i$

2 **a** $z = 3 \pm i\sqrt{7}$ **b** $z = 7 \pm 2i\sqrt{3}$ **c** $z = -1 \pm \frac{3}{4}i$

3 **a** $z = -1 \pm 2i$ **b** $z = 1 \pm 3i$ **c** $z = -2 \pm 5i$
d $z = -5 \pm i$ **e** $z = -\frac{5}{2} \pm \frac{5i\sqrt{3}}{2}$ **f** $z = \frac{-3 \pm i\sqrt{11}}{2}$

4 **a** $z = -\frac{5}{4} \pm \frac{\sqrt{7}}{4}i$ **b** $z = \frac{3}{14} \pm \frac{5\sqrt{3}}{14}i$ **c** $z = \frac{1}{10} \pm \frac{\sqrt{59}}{10}i$

5 $z_1 = 4 + \sqrt{5}\,i$ and $z_2 = 4 - \sqrt{5}\,i$

6 $-\sqrt{44} < b < \sqrt{44}$ or $-2\sqrt{11} < b < 2\sqrt{11}$

Exercise C

1 **a** $11 + 23i$ **b** $36 + 33i$ **c** $15 + 23i$ **d** $2 - 110i$
e $-5 - 25i$ **f** $39 + 80i$ **g** $-77 - 36i$ **h** $10i$
i $54 - 62i$ **j** $-46 + 9i$

2 **a** 41 **b** 53 **c** They are both real
d $(a + bi)(a - bi) = a^2 + b^2$, which is real.

3 $a = 7, b = -6$ or $a = 18, b = -\frac{7}{3}$

4 **a** -1 **b** 81 **c** $2i$ **d** $-60i$

5 $-8i, a = 0, b = -8$

6 $-119 - 120i$, so real part is -119

7 **a** $-2i$ **b** $-49 - 66i$

8 Substitute $z = 1 - 4i$ into $f(z)$ to get $f(z) = 0$.

9 **a** $i^3 = -i, i^4 = 1$ **b** $i^5 = i, i^6 = -1, i^7 = -i, i^8 = 1$
c **i** 1 **ii** i **iii** i

Exercise D

1 **a** $8-2\mathrm{i}$ **b** $6+5\mathrm{i}$ **c** $\frac{2}{3}+\frac{1}{2}\mathrm{i}$ **d** $\sqrt{5}-\mathrm{i}\sqrt{10}$

2 **a** $z+z^*=12, zz^*=45$ **b** $z+z^*=20, zz^*=125$

c $z+z^*=\frac{3}{2}, zz^*=\frac{5}{8}$ **d** $z+z^*=2\sqrt{5}, zz^*=50$

3 **a** $-\frac{6}{5}-\frac{7}{5}\mathrm{i}$ **b** $-\frac{11}{50}+\frac{27}{50}\mathrm{i}$ **c** $\frac{31}{2}+\frac{25}{2}\mathrm{i}$ **d** $\frac{6}{17}-\frac{7}{17}\mathrm{i}$

4 $-\frac{31}{2}-\frac{17}{2}\mathrm{i}$

5 **a** $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ **b** $\frac{7}{2}+\frac{1}{2}\mathrm{i}$ **c** $\frac{41}{5}-\frac{3}{5}\mathrm{i}$

6 $\frac{8}{5}+\frac{9}{5}\mathrm{i}$

7 $6+8\mathrm{i}$

8 $\dfrac{4}{8-\sqrt{2}\mathrm{i}}\times\dfrac{8+\sqrt{2}\mathrm{i}}{8+\sqrt{2}\mathrm{i}}=\dfrac{32+4\mathrm{i}\sqrt{2}}{66}=\dfrac{16}{33}+\dfrac{2\sqrt{2}}{33}\mathrm{i}$

9 $\dfrac{1}{1-9\mathrm{i}}\times\dfrac{1+9\mathrm{i}}{1+9\mathrm{i}}=\dfrac{1}{82}+\dfrac{9}{82}\mathrm{i}$

10 $\dfrac{z+4}{z-3}=\dfrac{(8-\mathrm{i}\sqrt{2})(1+\mathrm{i}\sqrt{2})}{(1-\mathrm{i}\sqrt{2})(1+\mathrm{i}\sqrt{2})}=\dfrac{10}{3}+\dfrac{7\sqrt{2}}{3}\mathrm{i}$

11 $z-2\mathrm{i}=\dfrac{(6-4\mathrm{i})(4-2\mathrm{i})}{(4+2\mathrm{i})(4-2\mathrm{i})}=\dfrac{16-28\mathrm{i}}{20}$

$z-2\mathrm{i}=\frac{4}{5}-\frac{7}{5}\mathrm{i}\Rightarrow z=\frac{4}{5}+\frac{3}{5}\mathrm{i}$

12 $\dfrac{(p-7\mathrm{i})(2-5\mathrm{i})}{(2+5\mathrm{i})(2-5\mathrm{i})}=\dfrac{2p-35}{29}+\dfrac{-5p-14}{29}\mathrm{i}$

13 $\dfrac{z}{z^*}=\dfrac{\sqrt{5}+4\mathrm{i}}{\sqrt{5}-4\mathrm{i}}\times\dfrac{\sqrt{5}+4\mathrm{i}}{\sqrt{5}+4\mathrm{i}}=-\dfrac{11}{21}+\dfrac{8\sqrt{5}}{21}\mathrm{i}$

14 **a** $\dfrac{p+5\mathrm{i}}{p-2\mathrm{i}}\times\dfrac{p+2\mathrm{i}}{p+2\mathrm{i}}=\dfrac{p^2-10+7p\mathrm{i}}{p^2+4}$

$2p^2-20=p^2+4$

$p^2=24\Rightarrow p=2\sqrt{6}$

b $\frac{1}{2}+\dfrac{\sqrt{6}}{2}\mathrm{i}$

14 The complex number z is defined by $z=\dfrac{p+5\mathrm{i}}{p-2\mathrm{i}}, p\in\mathbb{R}, p>0$.
Given that the real part of z is $\frac{1}{2}$,

a find the value of p **(4 marks)**

b write z in the form $a+b\mathrm{i}$, where a and b are real. **(1 mark)**

Exercise E

1 **a** $-1+5i, -1-5i$ **b** -2 **c** 26
2 **a** $4+3i, 4-3i$ **b** 8 **c** 25
3 **a** $2-3i$ **b** $z^2-4z+13=0$
4 **a** $5+i$
b $(z-(5-i))(z-(5+i))=0$
$z^2-(5+i)z-(5-i)z+(5-i)(5+i)=0$
$z^2-10z+26=0 \Rightarrow p=-10, q=26$
5 $z^2+10z+41=0$
6 $z^2-2z+5=0$
7 $z^2-6z+34=0$
8 **a** $z=\frac{3}{2}+\frac{1}{2}i$
b $\left(z-\left(\frac{3}{2}+\frac{1}{2}i\right)\right)\left(z-\left(\frac{3}{2}-\frac{1}{2}i\right)\right)$
$=z^2-z\left(\frac{3}{2}-\frac{1}{2}i\right)-z\left(\frac{3}{2}+\frac{1}{2}i\right)+\left(\frac{3}{2}-\frac{1}{2}i\right)\left(\frac{3}{2}+\frac{1}{2}i\right)$
$=z^2-3z+\frac{5}{2}$ so $p=-3, q=\frac{5}{2}$
9 $(z-(5+qi))(z-(5-qi))=z^2-10z+25+q^2$
$\Rightarrow 4p=10 \Rightarrow p=\frac{5}{2} \Rightarrow 25+q^2=34 \Rightarrow q=3$

Exercise F

1 **a** $f(2)=8-24+42-26=0$
b $z=2, z=2+3i$ or $z=2-3i$
2 **a** Substitute $z=\frac{1}{2}$ into $f(z)$.
b $b=3, c=6$
c $z=\frac{1}{2}$, or $z=-\frac{3}{2}\pm\frac{\sqrt{15}}{2}i$
3 $3, -\frac{1}{2}+\frac{1}{2}i$ and $-\frac{1}{2}-\frac{1}{2}i$
4 **a** $(z-(-4+i))(z-(-4-i))=z^2+8z+16+1$
$=z^2+8z+17$
b $z=4, z=-4+i$ or $z=-4-i$
5 **a** $a=8, b=25$ **b** $-1, -4+3i, -4-3i$ **c** -9
6 **a** $3-i$ **b** $c=46, d=-60$
7 $-\frac{1}{2}, -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ and $-\frac{1}{2}-\frac{\sqrt{3}}{2}i$
8 **a** $k=-40$ **b** $2-4i, 2+4i$
9 $2, -2, 2i$ and $-2i$
10 **a** $(z^2-9)(z^2-12z+40)$ **b** $z=\pm3, 6\pm2i$
11 $-3+i, -3-i, 2+3i$ and $2-3i$
$(z-(2-3i))(z-(2+3i))=z^2-4z+13$
12 **a** $(z^2-4z+13)(z^2+bz+c)$
$=z^4-10z^3+71z^2+Qz+442$
$b=-6, c=34$
b $Q=-214$ **c** $z=2+3i, 2-3i, 3+5i$ or $3-5i$

Exercise G

1 **a** $6+i$ **b** $-6+12i$ **c** $50-22i$

2 $-2\sqrt{14} < b < 2\sqrt{14}$

3 $3+i\sqrt{3}, 3-i\sqrt{3}$

4 $(1+2i)^5$

$= 1^5 + 5(1)^4(2i) + 10(1)^3(2i)^2 + 10(1)^2(2i)^3 + 5(1)(2i)^4 + (2i)^5$

$= 1 + 10i + 40i^2 + 80i^3 + 80i^4 + 32i^5$

$= 1 + 10i - 40 - 80i + 80 + 32i$

$= 41 - 38i$

5 Substitute $z = 3 + i$ into $f(z)$ to get $f(z) = 0$.

6 **a** $4-2i$ **b** $-14-2i$ **c** $-1-i$

7 $\dfrac{(45-28i)(1-i\sqrt{3})}{(1+\sqrt{3}i)(1-\sqrt{3}i)} = \dfrac{45-28\sqrt{3}}{4} + \left(\dfrac{-45\sqrt{3}-28}{4}\right)i$

8 $\dfrac{4-7i}{3+i} = \dfrac{(4-7i)(3-i)}{(3+i)(3-i)} = \dfrac{12-25i+7i^2}{10} = \frac{1}{2} - \frac{5}{2}i$

9 **a** $\frac{3}{25} - \frac{4}{25}i$ **b** $\frac{-8}{5} - \frac{6}{5}i$

10 $\dfrac{z}{z^*} = \dfrac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \dfrac{a^2+2abi+b^2i^2}{a^2-b^2i^2}$

$= \dfrac{a^2-b^2}{a^2+b^2} + \left(\dfrac{2ab}{a^2+b^2}\right)i$

11 **a** $\dfrac{3+qi}{q-5i} \times \dfrac{q+5i}{q+5i} = \dfrac{3q-5q}{q^2+25} + \dfrac{q^2+15}{q^2+25}i$

$\dfrac{-2q}{q^2+25} = \frac{1}{13} \Rightarrow q^2 + 26q + 25 = 0 \Rightarrow q = -1, q = -25$

b $\frac{1}{13} + \frac{8}{13}i, \frac{1}{13} + \frac{64}{65}i$

12 $x + yi + 4i(x - yi) = -3 + 18i$

$(x+4y) + (4x+y)i = -3 + 18i$

$x + 4y = -3, 4x + y = 18 \Rightarrow x = 5, y = -2$

13 $\dfrac{(9+6i)(2+3i)}{(2-3i)(2+3i)} = \dfrac{18+39i+18i^2}{4-9i^2} = 3i$

14 $\dfrac{(q+3i)(4-qi)}{(4+qi)(4-qi)} = \dfrac{7q}{q^2+16} + \dfrac{12-q^2}{q^2+16}i$

15 **a** $6+2i$ **b** $z^2 - 12z + 40$

16 $k = 6, m = 4$

17 $z = 2+i, 2-i$ or -4

18 $z = -2, 1+3i$ or $1-3i$

19 **a** $k = 16$ **b** $-4i$ and -3

20 **a** $b = 4, c = 10$ **b** $z = 6, -1, -2+\sqrt{6}i$ or $-2-\sqrt{6}i$

21 $3-2i, 3+2i, i\sqrt{6}$ and $-i\sqrt{6}$

22 **a** $p = -18$ **b** $1, 4, -\frac{3}{2} + \dfrac{\sqrt{15}}{2}i$ and $-\frac{3}{2} - \dfrac{\sqrt{15}}{2}i$