Parametric Equations

Exercise A

1 Find the coordinates of the point on the curve $x = 5t^2,\ y = 10t$

(a) when $t = 6$, (b) when $t = -1$.

2 Find the coordinates of the point on the curve $x = 1 - \frac{1}{t},\ y = 1 + \frac{1}{t}$

(a) when $t = 3$, (b) when $t = -1$.

3 The parametric equations of a curve are $x = 2\cos t,\ y = 2\sin t$, for $0 \leqslant t < 2\pi$. What is the value of t at the point $(0, 2)$?

4 A curve is given by $x = 5\cos t,\ y = 2\sin t$ for $0 \leqslant t < 2\pi$. Find the value of t at the point $\left(-2\frac{1}{2}, \sqrt{3}\right)$.

5 Sketch the curve given by $x = t^2, y = \frac{1}{t}$ for $t > 0$.

6 Sketch the curve given by $x = 3\cos t, y = 2\sin t$ for $0 \leqslant t < 2\pi$.

7 Sketch the graph of $x = 3t^2, y = 6t$ for $-4 \leqslant t \leqslant 4$.

8 Sketch the locus given by $x = \cos^2 t, y = \sin^2 t$ for $0 \leqslant t < 2\pi$.

9 Find cartesian equations for curves with these parametric equations.

(a) $x = t^2, y = \frac{1}{t}$ (b) $x = 3t^2, y = 6t$ (c) $x = 2\cos t, y = 2\sin t$

10 Find cartesian equations for curves with these parametric equations.

(a) $x = \cos^2 t,\ y = \sin^2 t$ (b) $x = \cos^3 t, y = \sin^3 t$

(c) $x = 1 - \frac{1}{t},\ y = 1 + \frac{1}{t}$ (d) $x = 3t^2, y = 2t^3$

11 Show that parametric equations for a circle with centre (p, q) and radius r are $x = p + r\cos t,\ y = q + r\sin t$. Eliminate the parameter t to obtain the cartesian equation of the circle in the form $(x - p)^2 + (y - q)^2 = r^2$.

Exercise B

1 Find $\frac{dy}{dx}$ in terms of t for the following curves.

(a) $x = t^3, y = 2t$ (b) $x = \sin t, y = \cos t$

(c) $x = 2\cos t, y = 3\sin t$ (d) $x = t^3 + t, y = t^2 - t$

2 Find the gradient of the tangent on the following curves, at the specified values of t.

(a) $x = 3t^2$, $y = 6t$ when $t = 0.5$

(b) $x = t^3$, $y = t^2$ when $t = 2$

(c) $x = 1 - \frac{1}{t}$, $y = 1 + \frac{1}{t}$ when $t = 2$

(d) $x = t^2$, $y = \frac{1}{t}$ when $t = 3$

3 Find the gradient of the normal on the following curves, at the specified values of t.

(a) $x = 5t^2$, $y = 10t$ when $t = 3$

(b) $x = \cos^2 t$, $y = \sin^2 t$ when $t = \frac{1}{3}\pi$

(c) $x = \cos^3 t$, $y = \sin^3 t$ when $t = \frac{1}{6}\pi$

(d) $x = t^2 + 2$, $y = t - 2$ when $t = 4$

4 Show that the equation of the tangent to the curve $x = 3\cos t$, $y = 2\sin t$ when $t = \frac{3}{4}\pi$ is $3y = 2x + 6\sqrt{2}$.

5 (a) Find the gradient of the curve $x = t^3$, $y = t^2 - t$ at the point $(1,0)$.

(b) Hence find the equation of the tangent to the curve at this point.

6 A curve has parametric equations $x = t - \cos t$, $y = \sin t$. Find the equation of the tangent to the curve when $t = \pi$.

7 Find the equations of the tangents to these curves at the specified values.

(a) $x = t^2$, $y = 2t$ when $t = 3$

(b) $x = 5\cos t$, $y = 3\sin t$ when $t = \frac{11}{6}\pi$

8 Find the equations of the normals to these curves at the specified values.

(a) $x = 5t^2$, $y = 10t$ when $t = 3$

(b) $x = \cos t$, $y = \sin t$ when $t = \frac{2}{3}\pi$

9 (a) Find the equation of the normal to the hyperbola $x = 4t$, $y = \frac{4}{t}$ at the point $(8,2)$.

(b) Find the coordinates of the point where this normal crosses the curve again.

10 (a) Find the equation of the normal to the parabola $x = 3t^2$, $y = 6t$ at the point where $t = -2$.

(b) Find the coordinates of the point where this normal crosses the curve again.

Exercise C

1.

A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. **[5]**

(ii) Find the cartesian equation of the curve. **[3]**

(iii) State the set of values that x can take and hence sketch the curve. **[3]**

2.

A curve has parametric equations $x = \sec\theta,\ y = 2\tan\theta$.

(i) Given that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$, show that $\frac{dy}{dx} = 2\operatorname{cosec}\theta$. **[3]**

(ii) Verify that the cartesian equation of the curve is $y^2 = 4x^2 - 4$. **[2]**

3.

A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

(i) Find $\frac{dy}{dx}$ in terms of t, simplifying your answer. **[3]**

(ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. **[4]**

(iii) Find a cartesian equation of the curve. **[2]**

4.

A curve has parametric equations $x = e^{3t}, y = te^{2t}$.

(i) Find $\frac{dy}{dx}$ in terms of t. Hence find the exact gradient of the curve at the point with parameter $t = 1$. **[4]**

(ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. **[3]**

5.

(a) Find the gradient of the curve $x^2 + xy + y^2 = 3$ at the point $(-1, -1)$. **[4]**

(b) A curve C has parametric equations

$$x = 2t^2 - 1,\ y = t^3 + t.$$

(i) Find the coordinates of the point on C at which the tangent is parallel to the y-axis. **[3]**

(ii) Find the values of t for which x and y have the same rate of change with respect to t. **[3]**

Question 6 is on the next page.

6.

A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. **[2]**

(ii) Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

(iii) Find the value of t at the point where this normal meets the curve again. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$. **[3]**

7.

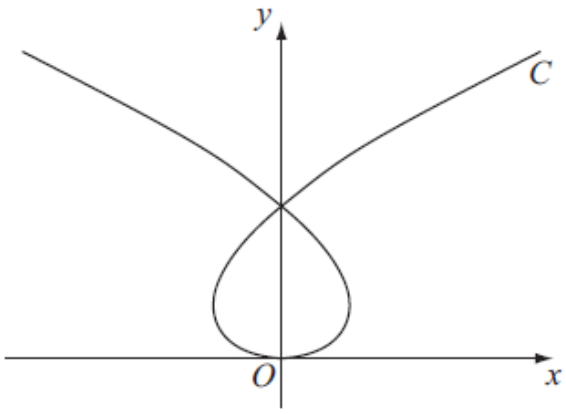


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

8.

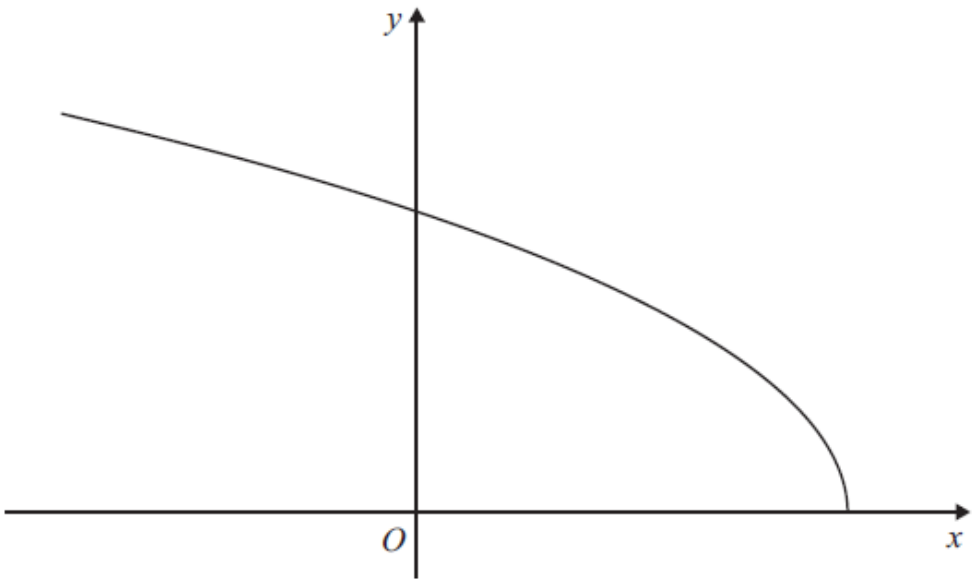


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(4)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

(Question 9 is on the next page.)

9.

The curve C has parametric equations

$$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \quad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \quad x > -4$$

where a and b are integers to be determined.

(3)

10.

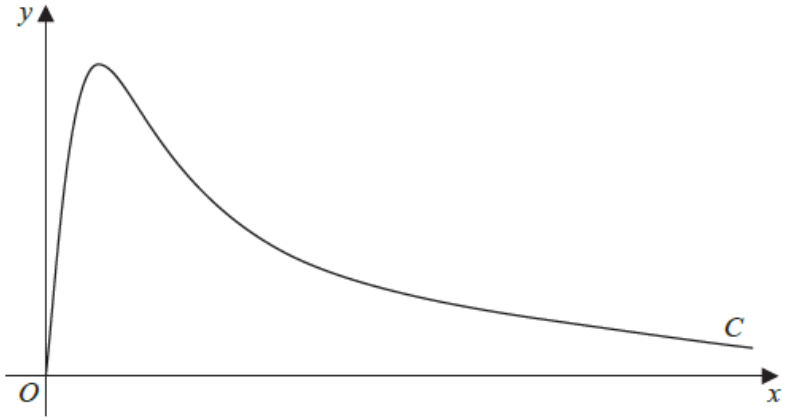


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q.

(2)