Answers - Capacitors - 1

Exercise A

1.

Charge $Q = CV = 20 \times 10^{-6}$ F $\times$ 6.0 V = 120 μC
Energy = $\frac{1}{2}QV = \frac{1}{2} \times 120 \times 10^{-6}$ C $\times$ 6.0 V = 360 μJ

When the voltage is doubled, the charge is doubled to 240 μC.

The energy stored is multiplied by 4, since both the charge and the voltage are doubled, to 1.44 mJ.

2.

F = C V^{-1}

= C $(JC^{-1})^{-1}$

= C $(J^{-1}$ C)

= C^2 J^{-1}

3.

4.5 V

4.

(a) 8.0 C;
(b) (i) 8.0×10^4 A; **(ii)** 1 6 MW

5.

(a) $Q = CV$ = 0.00100 μF × 12 V = 0.012 μC (Note how the μ symbol can be carried through the equation.)

(b) (i) $\frac{1}{C} = \frac{1}{0.00100} + \frac{1}{0.000020} = 1000 + 50000 = 51000$

$C = \frac{1}{51000} = 1.96 \times 10^{-5}$ μF

(ii) The charge Q on each capacitor and the total charge are the same, so

$Q = CV$ = 12 V × 1.96×10^{-5} μF = 2.35×10^{-4} μC

(iii) The p.d. across the 0.00100 μF capacitor is Q/C = 2.35×10^{-4} μC/0.00100 μF = 0.24 V. The p.d. across the 0.000020 μF capacitor is Q/C = 2.35×10^{-4} μC/0.000020 μF = 11.76 V

6.

(a) (i) 3.0 μF;
(ii) 180 μC, 90 μC, 90 μC;
(iii) 30 V across each
(b) 9.0 μF
(c) (i) 6.0 μF: 240 μC; 3.0 μF: 60 μC; 9.0 μF: 180 μC;
(ii) 6.0 μF: 40 V; 3.0 μF: 20 V; 9.0 μF: 20 V

7.

(a) (i) 15 mC; (ii) 10 mC
(b) 10 V
(c) (i) 0.11 J; (ii) 0.075 J

8.

The ratios are about 1.61 ± 0.05

9.

(a) $CR = 22.2 \times 10^{-6} \times 90.1 \times 10^{3} = 2.00$ s
(These figures were arranged to make the time constant equal to 2.00 s.)
(b) (i) $Q_0 = CV = 22.2 \times 10^{-6} \times 9.00 = 200\ \mu C$
$I_0 = -V_C/R = -9.00/90.1 \times 10^{3} = -99.9\ \mu A$
(the minus sign indicates discharge).
(c) (i) $Q = Q_0\, e^{-\frac{t}{CR}} = 200 \times 10^{-6}\, e^{-\frac{1.5}{2.0}} = 200 \times 10^{-6}\, e^{-0.75}$
$= 200 \times 10^{-6} \times 0.472 = 94.5\ \mu C$
(ii) $I = I_0\, e^{-\frac{t}{CR}} = -99.9 \times 10^{-6}\, e^{-\frac{1.5}{2.0}} = 99.9 \times 10^{-6}\, e^{-0.75}$
$= 99.9 \times 10^{-6} \times 0.472 = 47.2\ \mu A$
(d) $Q = Q_0\, e^{-\frac{CR}{CR}} = 200 \times 10^{-6}\, e^{-\frac{2.0}{2.0}} = 200 \times 10^{-6}\, e^{-1.0}$
$= 200 \times 10^{-6} \times 0.368 = 73.6\ \mu C$

The charge remaining on any capacitor after one time constant is always 36.8% of the charge at time $t = 0$.

10.

(a) 0.65 mA;
(b) (i) 6.5 V; (ii) 0.16 mA;
(c) (i) 9.6 V; (ii) 0.24 mA;
(e) 26 s.

Exercise B

1.

a i $Q_0 = CV_0 = 2200 \times 10^{-6} \times 9.0 = 2.0 \times 10^{-2}$ C.

ii Time constant $= RC = 100 \times 10^3 \times 2.2 \times 10^{-3} = 220$ s.

b i When $t = RC$, $V = V_0 e^{-1} = 0.37 \times 9.0 = 3.3$ V.

ii When $t = 300$ s, $\frac{t}{RC} = \frac{300}{220} = 1.36$

Therefore, $V = V_0 e^{\frac{-t}{RC}} = 9.0e^{-1.36} = 2.3$ V.

2.

a i 300 μC

ii 5.0 s

b i 5 s approx

ii 20 kΩ

3.

a i 0.61 mC

ii 0.45 mA

b 0.23 V, 11 μA

4.

a 13 μC, 40 μJ

b 0.62 V, 0.42 μJ

5.

a i 60 mA

ii 0.34 mJ

b 1.4 s

c 0.32 mJ

6.

a i *C* increases

ii *Q* increases

b The energy stored $= \frac{1}{2}QV$, so the energy stored increases because *Q* increases and *V* is unchanged.

7.

a 9.8 pF

b 1100 pJ