

Mixed Exercise - 3

1.

The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that **all** the terms in the new sequence are multiples of 5.

2.

Simplify $\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x}$

3.

$$x^{\frac{3}{2}} = 8 \text{ where } x > 0 \quad \text{and} \quad y^{-2} = \frac{25}{4} \text{ where } y > 0$$

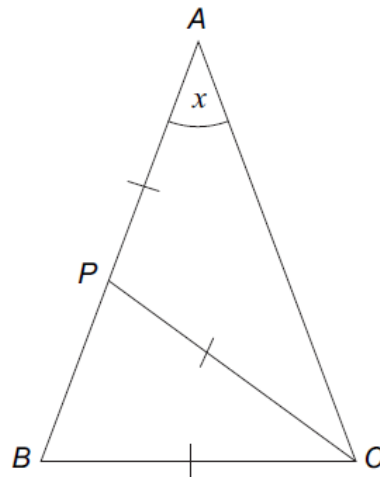
Work out the value of $\frac{x}{y}$.

4.

ABC is a triangle.

P is a point on AB such that $AP = PC = BC$

Angle $BAC = x$



Not drawn accurately

(a) Prove that angle $ABC = 2x$

- (b) You are also given that $AB = AC$

Work out the value of x .

5.

$$1 \leq m \leq 5 \quad \text{and} \quad -9 \leq n \leq 2$$

- (a) Work out an inequality for $m + n$.

Answer..... $\leq m + n \leq$

- (b) Work out an inequality for $(m + n)^2$.

Answer..... $\leq (m + n)^2 \leq$

6.

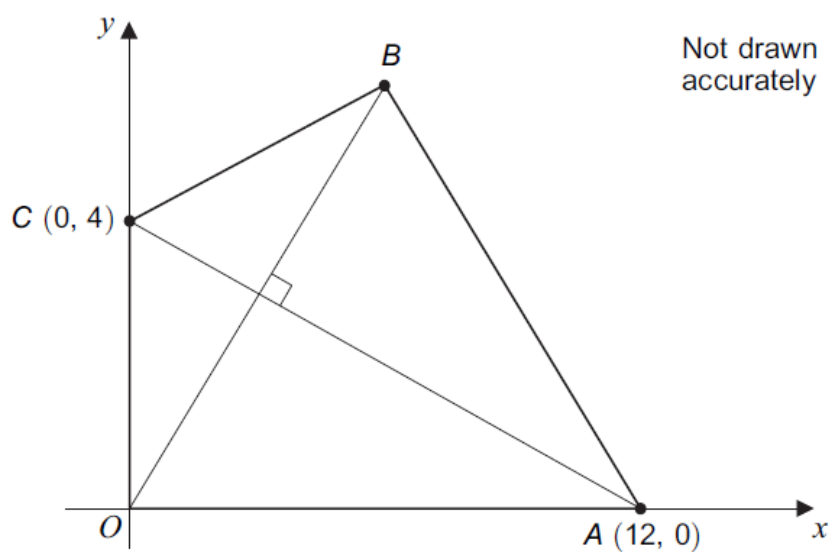
A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

7.

$OABC$ is a kite.



- (a) Work out the equation of AC .

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Answer..... (2 marks)

- (b) Work out the coordinates of B .

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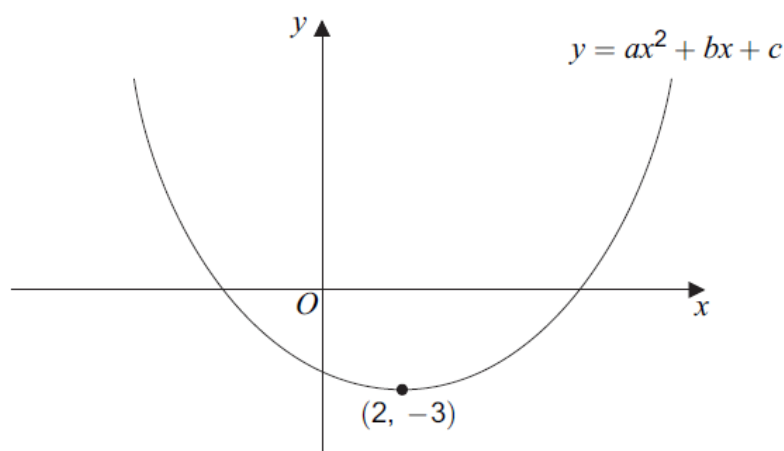
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Answer (..... ,) (6 marks)

8.

A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is $(2, -3)$.



For the sketch shown, circle the correct answer in each of the following.

- (a) The value of a is
- | | | | |
|------|----------|----------|----------|
| zero | positive | negative | (1 mark) |
|------|----------|----------|----------|
- (b) The value of c is
- | | | | |
|------|----------|----------|----------|
| zero | positive | negative | (1 mark) |
|------|----------|----------|----------|

- (c) The solutions of $ax^2 + bx + c = 0$ are
- both zero both positive both negative one positive and one negative
- (1 mark)
- (d) The **number** of solutions of $ax^2 + bx + c = -6$ is
- 0 1 2 3 (1 mark)
- (e) The equation of the tangent to $y = ax^2 + bx + c$ at $(2, -3)$ is
- $x = 2$ $y = 2$ $x = -3$ $y = -3$ (1 mark)
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9.

Solve the simultaneous equations

$$\begin{aligned}x + y &= 4 \\ y^2 &= 4x + 5\end{aligned}$$

Do **not** use trial and improvement.

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Answer..... (6 marks)

10.

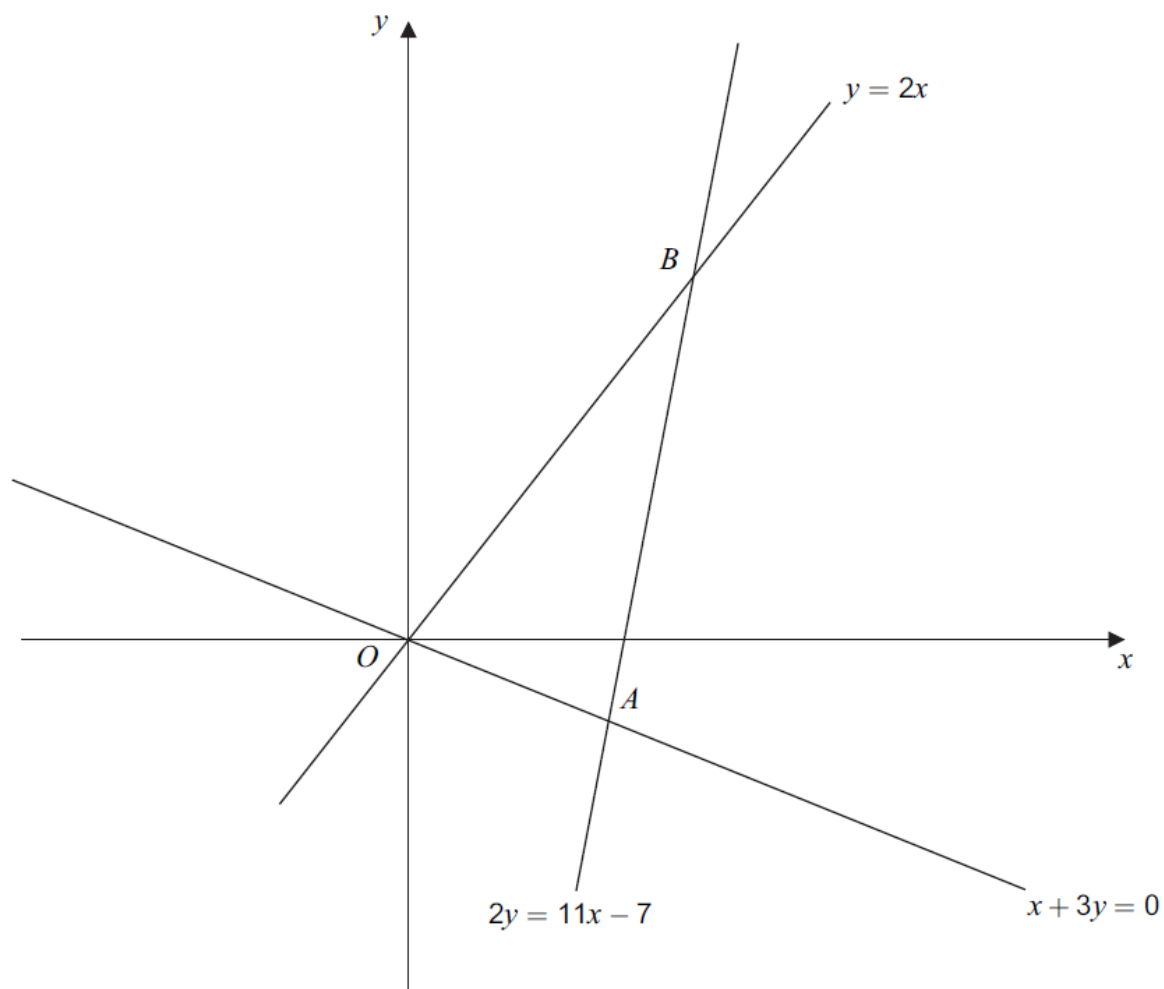
The equations of three straight lines are

$$y = 2x$$

$$x + 3y = 0$$

$$2y = 11x - 7$$

The lines intersect at the points O , A and B as shown on this sketch.



Show that $\text{length } OB = \text{length } AB$

