

Sequences and Series 2

1.

On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

- (a) Show that, immediately after his 12th birthday, the total of these gifts was £225 (1)

- (b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday. (2)

- (c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday. (3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

- (d) Show that $n^2 + 7n = 25 \times 18$ (3)

- (e) Find the value of n , when he had received £3375 in total, and so determine John's age at this time. (2)
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2.

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

- (a) Show that the shop sold 220 computers in 2007. (2)

- (b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive. (3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

- (c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred. (4)

3.

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N .

(a) Find the value of N .

(2)

The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

4.

(i) A sequence $U_1, U_2, U_3, \dots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

(ii) Another sequence $V_1, V_2, V_3, \dots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

(a) Find V_3 and V_4 in terms of k .

(2)

Given that $\sum_{n=1}^5 V_n = 165$,

(b) find the value of k .

(3)

5.

A sequence $a_1, a_2, a_3, \dots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geq 1$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k .

(2)

Find

(b) $\sum_{r=1}^3 (1 + a_r)$ in terms of k , giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100} (a_{r+1} + ka_r)$

(1)

6.

(a) A geometric series has first term a and common ratio r . Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$

(4)

Mr King will be paid a salary of £35 000 in the year 2005. Mr King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

(b) Find, to the nearest £100, Mr King's salary in the year 2008.

(2)

Mr King will receive a salary each year from 2005 until he retires at the end of 2024.

(c) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.

(4)

7.

A geometric series is $a + ar + ar^2 + \dots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \dots$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

8.

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r , $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000 r will be made.

(a) Write down an expression for the predicted profit in Year n .

(1)

The model predicts that in Year n , the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)