The Method of Iteration to Solve Equations

1.

Show that the following equations have a solution in the interval shown in the brackets.

(a) $x^3 - 2x^2 - 5 = 0$ $(2 < x < 3)$

(b) $2x^4 - 3x^2 - 1 = 0$ $(-2 < x < -1)$

(c) $x^5 - x - 2 = 0$ $(1 < x < 2)$

(d) $x^4 - 10x^2 + 2 = 0$ $(-4 < x < -3)$

(e) $x^3 + 2x^2 - 3x - 6 = 0$ $(1 < x < 2)$

(f) $x^6 - 7x^2 + 4 = 0$ $(0 < x < 1)$

2.

The equation $3x - 2x^3 + 5 = 0$ has one solution.

a) Show that this solution lies in the interval $1.5 < x < 2$.

b) Show that $3x - 2x^3 + 5 = 0$ can be written as:

$$x = \sqrt[3]{\frac{3x+5}{2}}$$

c) Use the iteration $x_{n+1} = \sqrt[3]{\frac{3x_n + 5}{2}}$ to find the solution to $3x - 2x^3 + 5 = 0$ to 5 d.p.
Use a starting value of $x_0 = 2$.

3.

a Show that the equation $x^6 - 5x + 3 = 0$ has a root between $x = 1$ and $x = 1.5$.

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b Use the iteration formula $x_{n+1} = \sqrt[5]{5 - \frac{3}{x_n}}$ to find an approximation for the root of the equation $x^6 - 5x + 3 = 0$, giving your answer to 2 decimal places.

4.

a Show that the equation $x^2 - 3x - 5 = 0$ can be rewritten in the form $x = \sqrt{3x + 5}$

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b Using $x = \sqrt{3x + 5}$ with $x_0 = 4$, use iteration to find one root of the equation $x^2 - 3x - 5 = 0$, giving your answer correct to 5 d.p.

5.

(a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \dfrac{5}{x^2 + 7}$

(2)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

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(2)

6.

(a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

(1)

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

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(3)

7.

Using $x_{n+1} = -2 - \frac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_1 =$ .....

$x_2 =$ .....

$x_3 =$ .....

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

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(2)

8.

Use iteration to find one root of each of these equations, giving your answers correct to 5 d.p.

a $x^2 - 4x - 4 = 0$, use $x_0 = 5$

b $x^2 - 6x - 8 = 0$, use $x_0 = 7$