Mixed Exercise 5

1.

Find the value of n such that $4^n \times 8^{n+1} = 16$
Show clear algebraic working.

2.

Simplify $\left(5h^{\frac{4}{3}}g^2\right)^3$

3.

Use algebra to show that the recurring decimal $0.0\dot{2}\dot{4} = \frac{4}{165}$

4.

$$M = \frac{b - c}{a}$$

$a = 5.3$ correct to 1 decimal place.
$b = 346.6$ correct to 1 decimal place.
$c = 80.0$ correct to 1 decimal place.

Calculate the upper bound for the value of M.
Show your working clearly.

5.

$(2x + 23)$, $(8x + 2)$ and $(20x - 52)$ are three consecutive terms of an arithmetic sequence.

Prove that the common difference of the sequence is 12

6.

Point A has coordinates $(4, 1)$
Point B has coordinates $(8, -2)$

A and B lie on the straight line **L**.

(a) Work out the gradient of **L**.

(2)

(b) Find an equation for **L**.
Give your answer in the form $ax + by = c$ where a, b and c are integers.

(3)

The straight line **M** is parallel to **L** and passes through the point $(0, 7)$

(c) Write down an equation for **M**.

(1)

7.

Line **L** has equation $4y - 6x = 33$
Line **M** goes through the point A (5, 6) and the point B (−4, k)

L is perpendicular to **M**.

Work out the value of k.

8.

Here is triangle ABC.

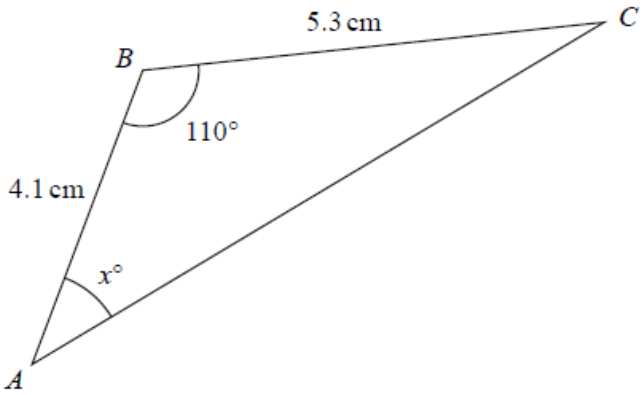


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 3 significant figures.

9.

The diagram shows a parallelogram *LMNP*.

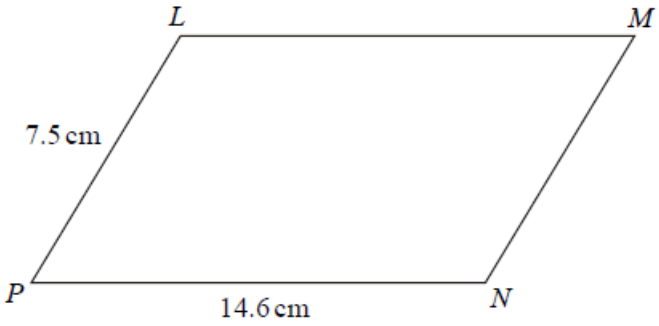


Diagram **NOT** accurately drawn

$LN = 13.3$ cm

Calculate the area of the parallelogram.
Give your answer correct to 3 significant figures.

10.

The diagram shows a sector *OABC* of a circle, centre *O* and radius 15 cm.

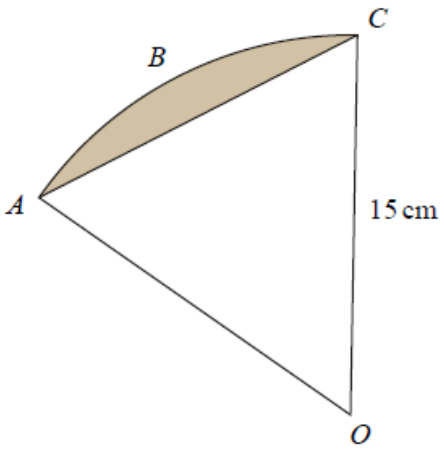


Diagram **NOT** accurately drawn

The length of arc $ABC = 3\pi$ cm.

Work out the area of the shaded segment.
Give your answer correct to 1 decimal place.

11.

The diagram shows a rectangular based pyramid.

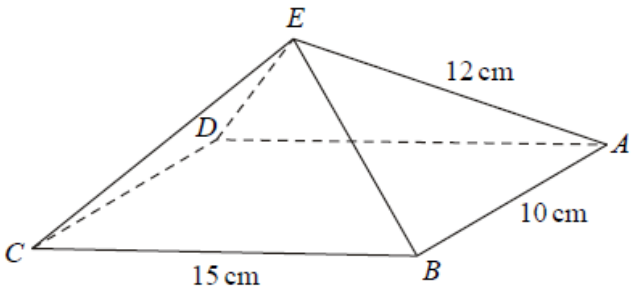


Diagram **NOT** accurately drawn

$AE = BE = CE = DE = 12$ cm
$AB = 10$ cm and $CB = 15$ cm

Calculate the size of angle CEA.
Give your answer correct to 1 decimal place.

12.

(a) Rationalise the denominator of $\frac{a+\sqrt{4b}}{a-\sqrt{4b}}$ where a is an integer and b is a prime number.

Simplify your answer.

(3)

(b) Given that $\left(\sqrt{\frac{y}{x}}\right)^{-5} = \frac{x^m}{y^m}$ where $x \neq y$

find the value of m.

13.

T is directly proportional to the cube of r

$T = 21.76$ when $r = 4$

(a) Find a formula for T in terms of r

(3)

(b) Work out the value of T when $r = 6$

(1)

14.

p is directly proportional to the cube of w

$p = 37.5$ when $w = 5$

(a) Find a formula for p in terms of w

(3)

(b) Calculate the value of p when $w = 4$

(1)

15.

Here is a solid shape **S**.

The solid shape consists of a cylinder and a hemisphere.
The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

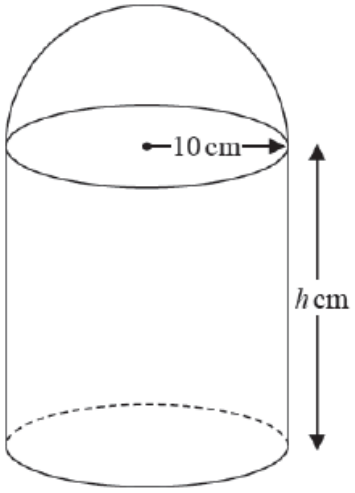


Diagram **NOT** accurately drawn

The radius of the cylinder and the radius of the hemisphere are both 10 cm.
The height of the cylinder is h cm.

The total surface area of **S** is 1000π cm^2

Find the value of h.

16.

The diagram shows a solid cylinder.

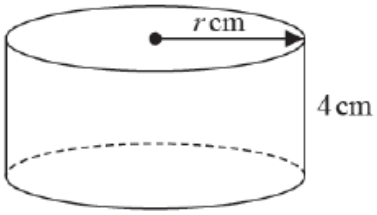


Diagram **NOT** accurately drawn

The cylinder has height 4 cm and radius r cm.

The total area of the two circular faces of the cylinder is 10π cm^2 greater than the curved surface area of the cylinder.

Work out the value of r.

17.

The diagram shows a trapezium.

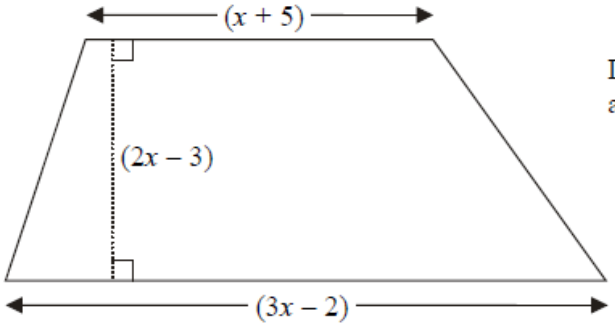


Diagram **NOT** accurately drawn

All measurements shown on the diagram are in centimetres.

The area of the trapezium is 133 cm^2

(a) Show that $8x^2 - 6x - 275 = 0$

(b) Find the value of x.
Show your working clearly.

$x =$

(3)

18.

Henry puts 8 coins in a bag.
The table gives information about the value of the coins.

Value of coin	5p	2p	1p
Number of coins	5	2	1

Henry then takes at random two coins from the bag.

(a) Work out the probability that the two coins are both 5p coins.

(2)

(b) Work out the probability that the total value of the two coins is at least 6p.

(3)

19.

A bowl contains n pieces of fruit.
Of these, 4 are oranges and the rest are apples.

Two pieces of fruit are going to be taken at random from the bowl.

The probability that the bowl will then contain $(n - 6)$ apples is $\frac{1}{3}$

Work out the value of n
Show your working clearly.

20.

Solve the simultaneous equations

$$y = 8 - 2x$$

$$x^2 + y^2 = 29$$

Show clear algebraic working.

21.

Solve

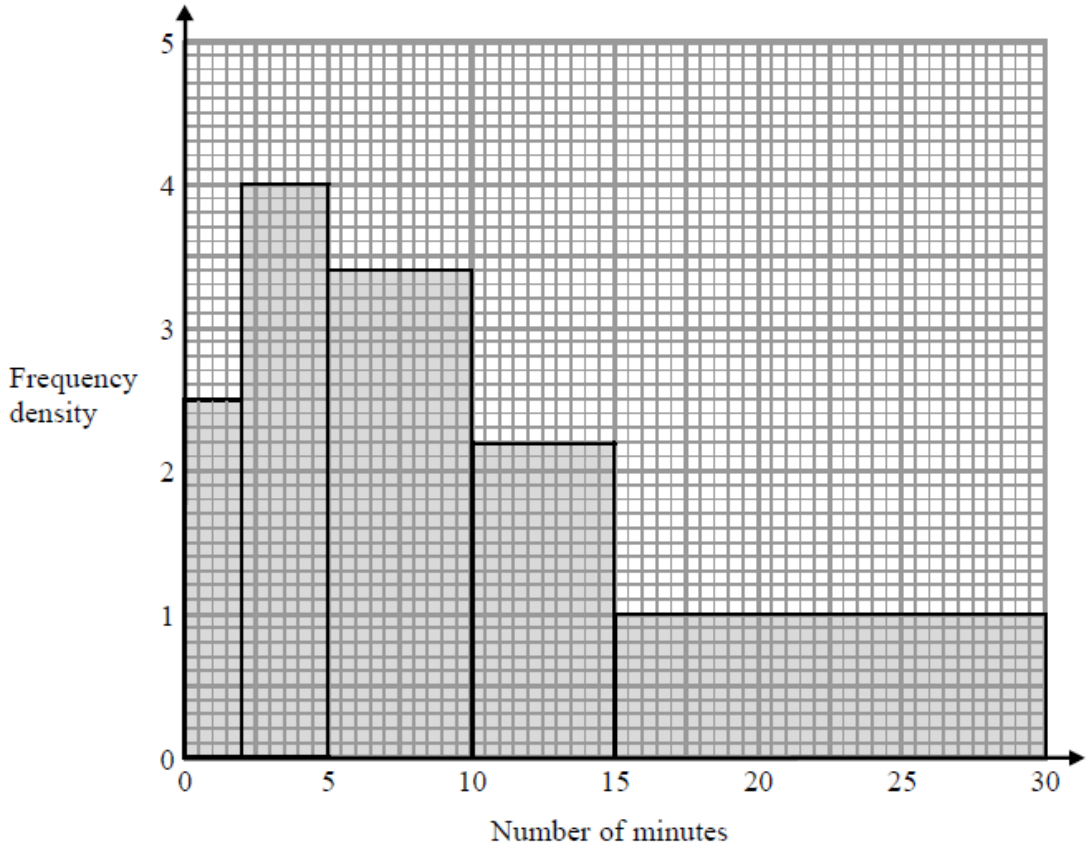
$$2x^2 + y^2 = 15$$

$$x = y - 3$$

Show your working clearly.
Give your solutions correct to 3 decimal places.

22.

The histogram shows information about the numbers of minutes some people waited to be served at a Post Office.

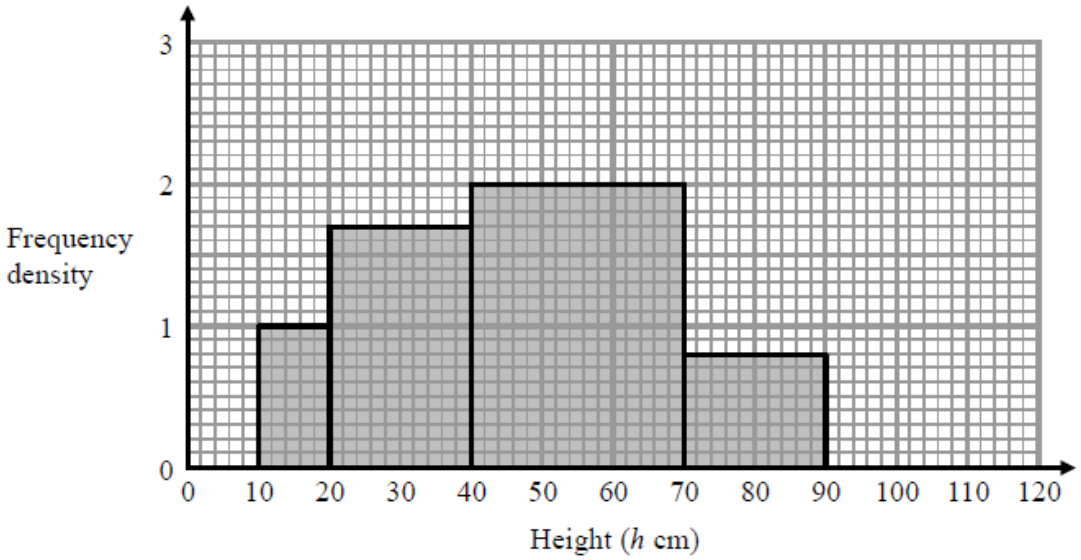


Work out an estimate for the proportion of these people who waited longer than 20 minutes to be served.

23.

Mayang collected bamboo plants for an experiment.
The heights of Mayang's bamboo plants are all between 10 cm and 110 cm.

The incomplete histogram gives some information about the heights, h cm, of the bamboo plants.



Mayang found that 4% of the bamboo plants had heights in the interval $90 < h \leqslant 110$

Use this information to complete the histogram.

24.

The table shows information about the marks gained by 200 students in a music examination.

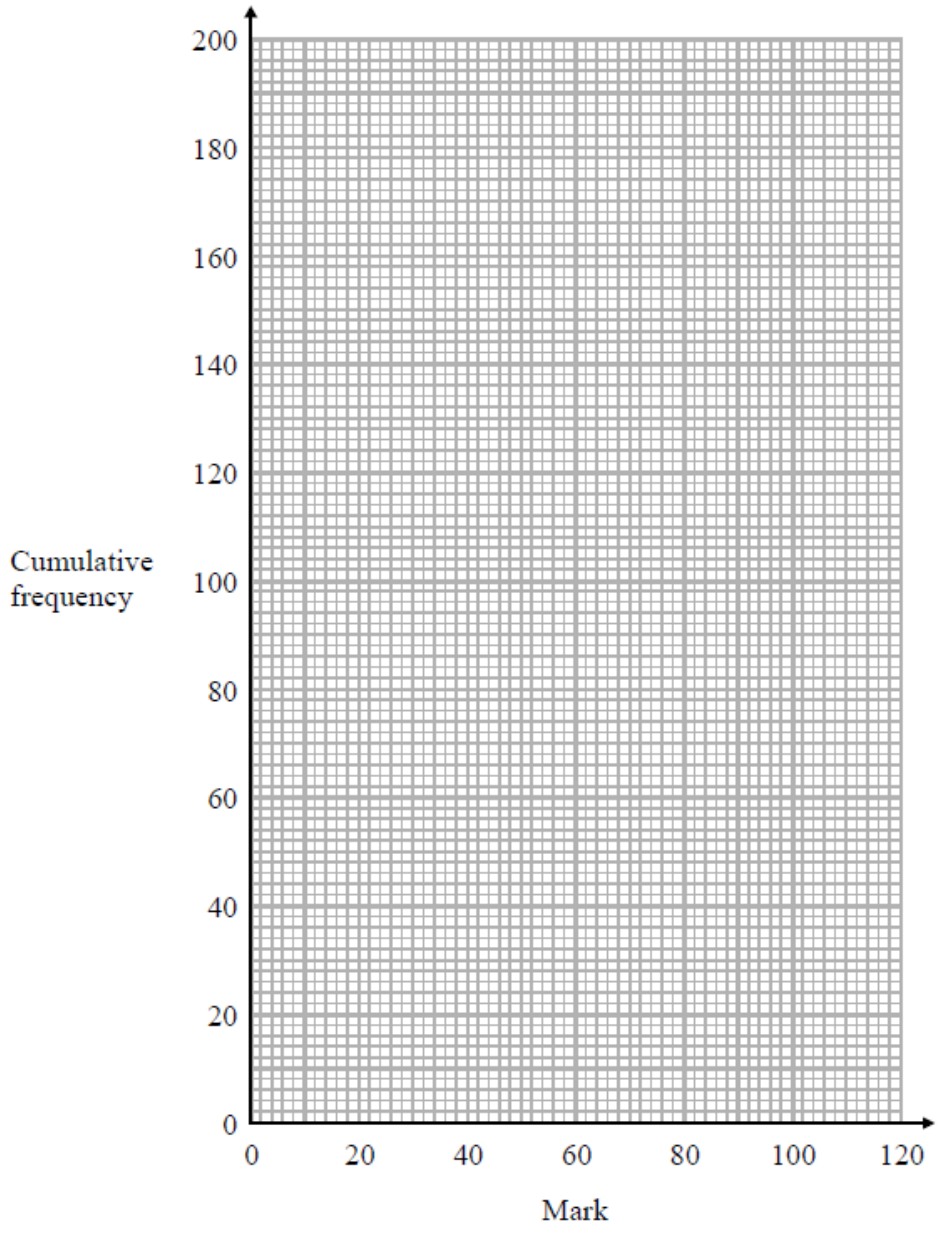
Mark (m)	Frequency
$0 < m \leqslant 20$	15
$20 < m \leqslant 40$	25
$40 < m \leqslant 60$	80
$60 < m \leqslant 80$	50
$80 < m \leqslant 100$	20
$100 < m \leqslant 120$	10

(a) Complete the cumulative frequency table.

Mark (m)	Cumulative frequency
$0 < m \leqslant 20$	
$0 < m \leqslant 40$	
$0 < m \leqslant 60$	
$0 < m \leqslant 80$	
$0 < m \leqslant 100$	
$0 < m \leqslant 120$	

(1)

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the median mark.

(1)

(d) Use your graph to find an estimate for the number of students who gained more than 75 marks.

(2)

25.

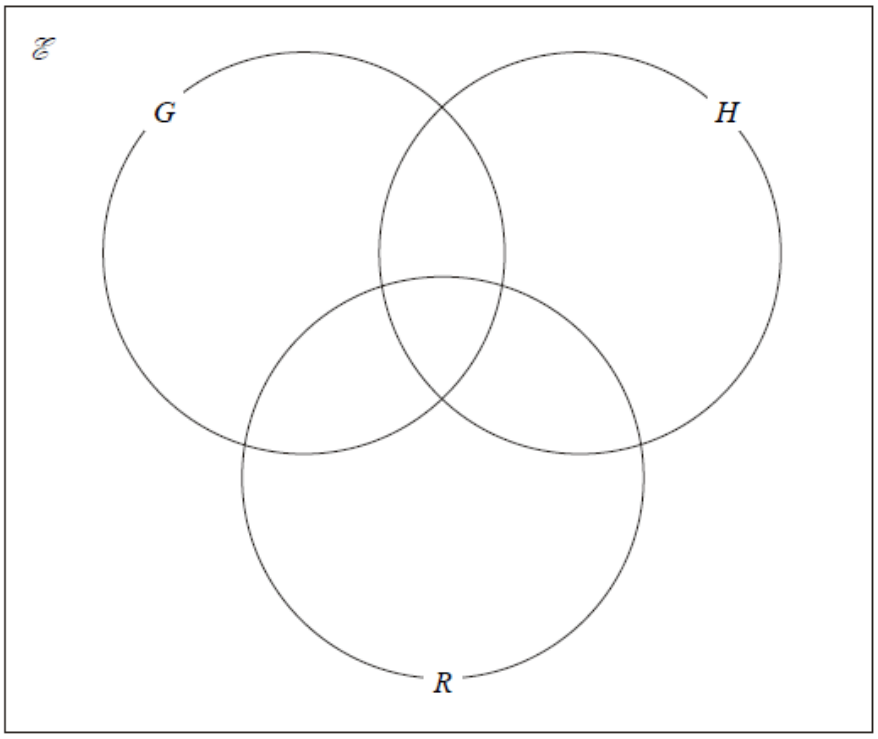
All the students in Year 11 at a school must study at least one of Geography (G), History (H) and Religious Studies (R).

In Year 11 there are 65 students.

Of these students

15 study Geography, History and Religious Studies
21 study Geography and History
16 study Geography and Religious Studies
30 study Geography
18 study only Religious Studies
37 study Religious Studies

(a) Using this information, complete the Venn diagram to show the number of students in each region of the Venn diagram.



(3)

A student in Year 11 who studies both History and Religious Studies is chosen at random.

(b) Work out the probability that this student does **not** study Geography.

(2)

26.

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$
$B = \{\text{numbers greater than } 6\}$

(a) List the members of the set $A \cup B$

(1)

$C = \{3, 6, 9\}$

(b) List the members of the set $A \cap C$

(1)

D is a set with 4 members.

$5 \in D$ and $B \cap D = \varnothing$

(c) List the members of one possible set D.

(2)

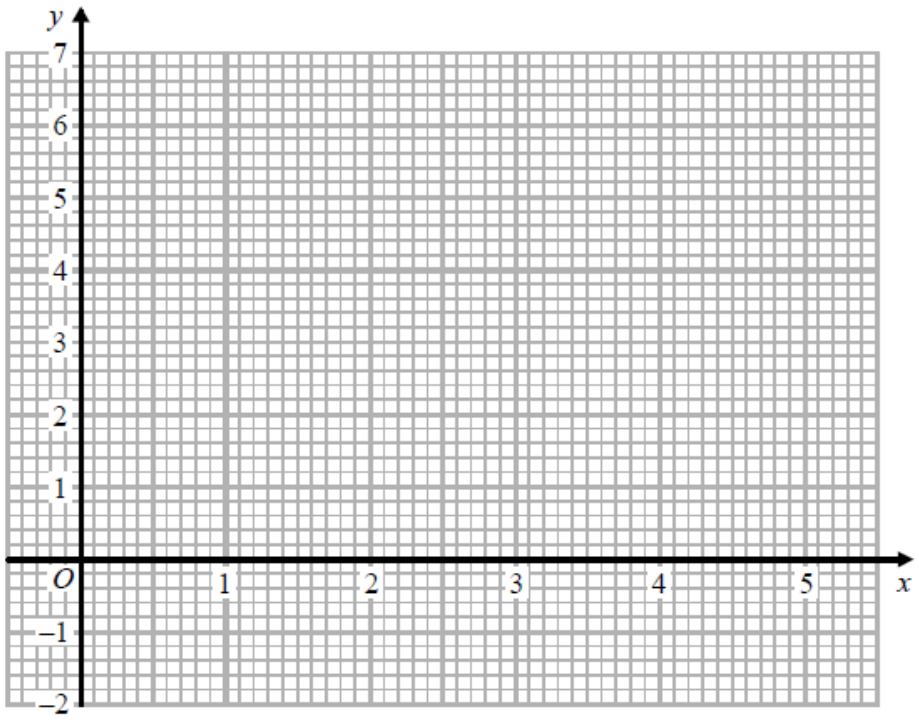
27.

(a) Complete the table of values for $y = x^2 - 5x + 6$

x	0	1	2	3	4	5
y	6		0	0	2	

(1)

(b) On the grid, draw the graph of $y = x^2 - 5x + 6$ for $0 \leqslant x \leqslant 5$



(2)

(c) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation

$$x^2 - 5x = x - 7$$

28.

ABC is an isosceles triangle such that

$AB = AC$
A has coordinates (4, 37)
B and C lie on the line with equation $3y = 2x + 12$

Find an equation of the line of symmetry of triangle ABC.
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show clear algebraic working.

29.

The diagram shows an isosceles triangle.

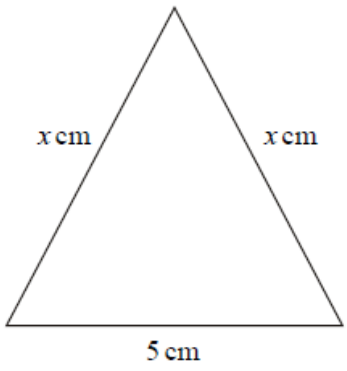


Diagram **NOT** accurately drawn

The area of the triangle is 12 cm^2

Work out the perimeter of the triangle.
Give your answer correct to 3 significant figures.

30.

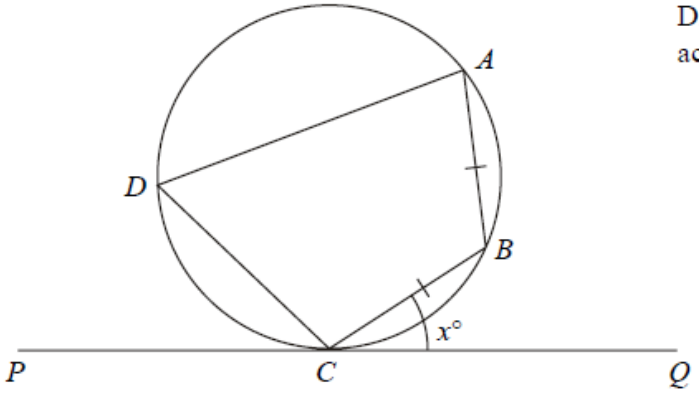


Diagram **NOT** accurately drawn

A, B, C and D are points on a circle.
PCQ is a tangent to the circle.
$AB = CB$.

Angle $BCQ = x°$

Prove that angle $CDA = 2x°$
Give reasons for each stage in your working.

31.

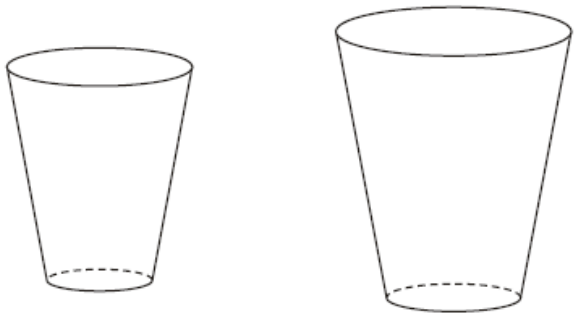
$$P = \frac{a}{m - x}$$

$x = 8$ correct to 1 significant figure
$a = 4.6$ correct to 2 significant figures
$m = 20$ correct to the nearest 10

Calculate the lower bound of P.
Show your working clearly.

32.

The diagram shows two mathematically similar vases, **A** and **B**.



A **B**

Diagram **NOT** accurately drawn

A has a volume of 405 cm^3
B has a volume of 960 cm^3

B has a surface area of 928 cm^2

Work out the surface area of **A**.

33.

The straight line **L** passes through the point with coordinates $(6, -4)$ and is parallel to the straight line with equation $y = 5 - 3x$

Find an equation for **L**.

34.

The diagram shows a solid cone.

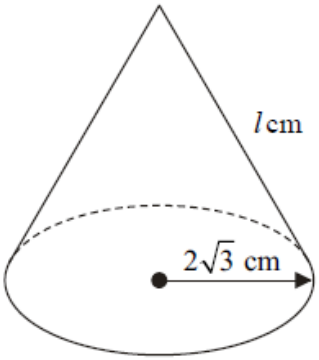


Diagram **NOT** accurately drawn

The radius of the base of the cone is $2\sqrt{3}$ cm.
The slant height of the cone is l cm.
The **total** surface area of the cone is 36π cm^2

Work out the exact value of l.

Give your answer in the form $2\sqrt{a}$ where a is an integer.