

Exam Questions – Set 6

1.

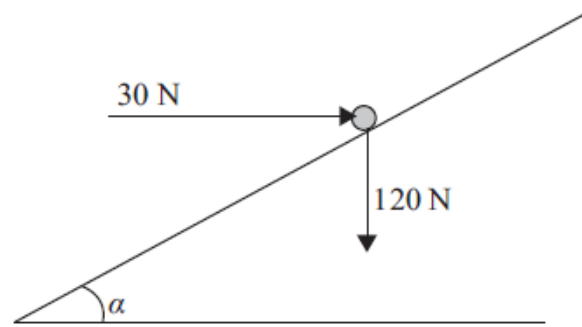


Figure 2

A particle of weight 120 N is placed on a fixed rough plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The particle is held at rest in equilibrium by a horizontal force of magnitude 30 N, which acts in the vertical plane containing the line of greatest slope of the plane through the particle, as shown in Figure 2.

- (a) Show that the normal reaction between the particle and the plane has magnitude 114 N. (4)

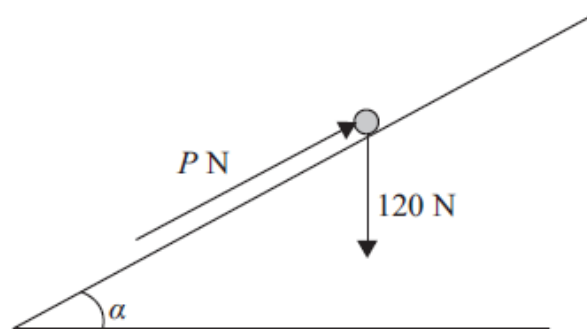


Figure 3

The horizontal force is removed and replaced by a force of magnitude P newtons acting up the slope along the line of greatest slope of the plane through the particle, as shown in Figure 3. The particle remains in equilibrium.

- (b) Find the greatest possible value of P . (8)
- (c) Find the magnitude and direction of the frictional force acting on the particle when $P = 30$. (3)

2.

The velocity $v \text{ m s}^{-1}$ of a car at time $t \text{ s}$, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is 1.3 m s^{-2} . For $t > 20$ the car continues its journey with constant acceleration 1.3 m s^{-2} until its speed reaches 25 m s^{-1} .

(i) Find the value of k . [3]

(ii) Find the total distance the car has travelled when its speed reaches 25 m s^{-1} . [7]

3.

A random variable X has probability distribution defined as follows.

$$P(X = x) = \begin{cases} kx & x = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $P(X = 3) = 0.2$. [3]

(b) Show in a table the values of X and their probabilities. [2]

(c) Two independent values of X are chosen, and their total T is found.

(i) Find $P(T = 7)$. [3]

(ii) Given that $T = 7$, determine the probability that one of the values of X is 2. [4]

4.

It is known that the lifetime of a certain species of animal in the wild has mean 13.3 years. A zoologist reads a study of 50 randomly chosen animals of this species that have been kept in zoos. According to the study, for these 50 animals the sample mean lifetime is 12.48 years and the population variance is 12.25 years^2 .

Test at the 5% significance level whether these results provide evidence that animals of this species that have been kept in zoos have a shorter expected lifetime than those in the wild. [7]

Question 5 is on the next page.

5.

Yuto works in the quality control department of a large company. The time, T minutes, it takes Yuto to analyse a sample is normally distributed with mean 18 minutes and standard deviation 5 minutes.

- (a) Find the probability that Yuto takes longer than 20 minutes to analyse the next sample. (3)

The company has a large store of samples analysed by Yuto with the time taken for each analysis recorded. Serena is investigating the samples that took Yuto longer than 15 minutes to analyse.

She selects, at random, one of the samples that took Yuto longer than 15 minutes to analyse.

- (b) Find the probability that this sample took Yuto more than 20 minutes to analyse. (4)

Serena can identify, in advance, the samples that Yuto can analyse in under 15 minutes and in future she will assign these to someone else.

- (c) Estimate the median time taken by Yuto to analyse samples in future. (5)
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6.

[In this question \mathbf{i} and \mathbf{j} are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O .]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ km h}^{-1}$ and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ km h}^{-1}$.

- (a) Find, to the nearest degree, the bearing on which Q is moving. (2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j}) \text{ km}$ and ship Q is at the point with position vector $(-2\mathbf{j}) \text{ km}$.

At time t hours after 2 pm, the position vector of P is $\mathbf{p} \text{ km}$ and the position vector of Q is $\mathbf{q} \text{ km}$.

- (b) Write down expressions, in terms of t , for

(i) \mathbf{p} ,

(ii) \mathbf{q} ,

(iii) \overrightarrow{PQ} .

(5)

(c) Find the time when

- (i) Q is due north of P ,
- (ii) Q is north-west of P .

(4)
