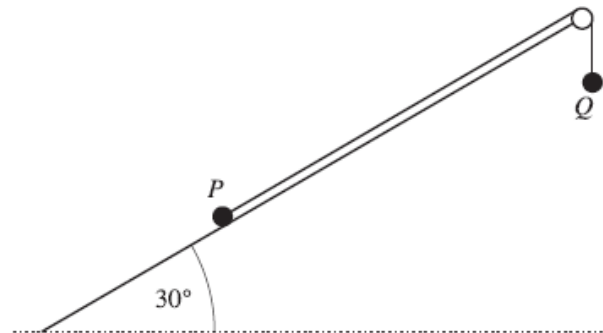# Exam Questions – Set 5

1.

Two particles P and Q are attached to opposite ends of a light inextensible string which passes over a small smooth pulley at the top of a rough plane inclined at 30° to the horizontal. P has mass 0.2 kg and is held at rest on the plane. Q has mass 0.2 kg and hangs freely. The string is taut (see diagram). The coefficient of friction between P and the plane is 0.4. The particle P is released.

  (i) State the tension in the string before P is released, and find the tension in the string after P is released.

[6]

Q strikes the floor and remains at rest. P continues to move up the plane for a further distance of 0.8 m before it comes to rest. P does not reach the pulley.

  (ii) Find the speed of the particles immediately before Q strikes the floor.

[5]

  (iii) Calculate the magnitude of the contact force exerted on P by the plane while P is in motion.

[3]

2.

*[In this question, **i** and **j** are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]*

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

3.

A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i)   the value of d,

(ii) the value of M.

(7)

4.

Mr McGregor is a keen vegetable gardener. A pigeon that eats his vegetables is his great enemy.

One day he sees the pigeon sitting on a small branch of a tree. He takes a stone from the ground and throws it. The trajectory of the stone is in a vertical plane that contains the pigeon. The same vertical plane intersects the window of his house. The situation is illustrated in Fig. 5.
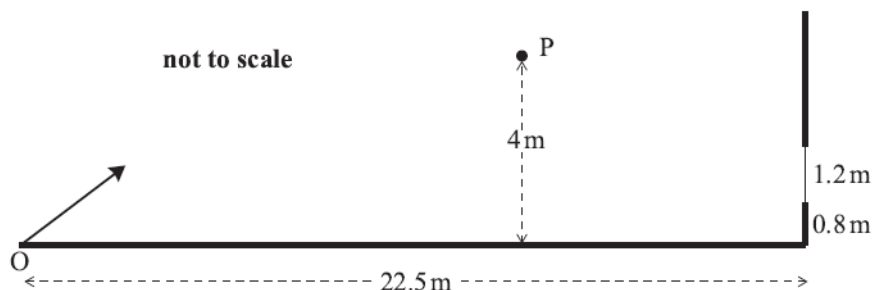

not to scale

P

4 m

1.2 m

0.8 m

O

22.5 m

Fig. 5

- The stone is thrown from point O on level ground. Its initial velocity is $15\,\mathrm{ms}^{-1}$ in the horizontal direction and $8\,\mathrm{ms}^{-1}$ in the vertical direction.
- The pigeon is at point P which is 4 m above the ground.
- The house is 22.5 m from O.
- The bottom of the window is 0.8 m above the ground and the window is 1.2 m high.

Show that the stone does not reach the height of the pigeon.

Determine whether the stone hits the window.

[7]

5.

The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.

(a) Find the proportion of men that take longer than 300 minutes to run a marathon.

(3)

Nathaniel is preparing to run a marathon. He aims to finish in the first 20% of male runners.

(b) Using the above model estimate the longest time that Nathaniel can take to run the marathon and achieve his aim.

(3)

The time, W minutes, taken by women to run a marathon is modelled by a normal distribution with mean μ minutes.

Given that $P(W < \mu + 30) = 0.82$

(c) find $P(W < \mu - 30 \mid W < \mu)$

(3)

6.

Mrs George claims that 45% of voters would vote for her.

In an opinion poll of 20 randomly selected voters it was found that 5 would vote for her.

(a)

Test at the 5% level of significance whether or not the opinion poll provides evidence to support Mrs George's claim.

(4)

In a second opinion poll of n randomly selected people it was found that no one would vote for Mrs George.

(b)

Using a 1% level of significance, find the smallest value of n for which the hypothesis $H_0 : p = 0.45$ will be rejected in favour of $H_1 : p < 0.45$
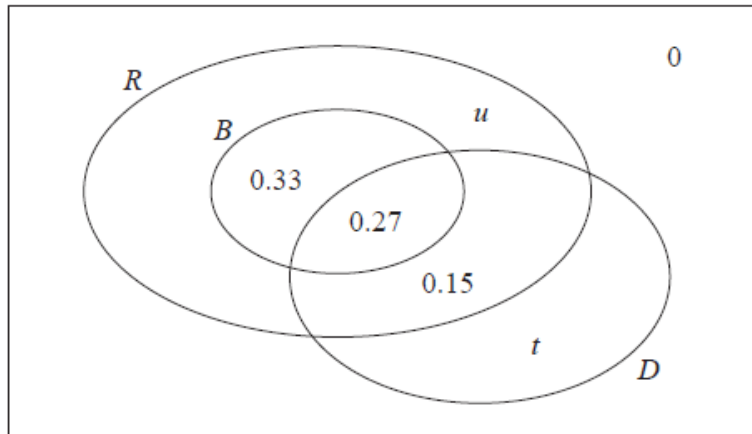
(3)

7.

The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room

B is the event that a customer books breakfast

D is the event that a customer books dinner

u and t are probabilities.

(a) Write down the probability that a customer books breakfast but does not book a room.

(1)

Given that the events B and D are independent

(b) find the value of t

(4)

(c) hence find the value of u

(2)

(d) Find

(i) $P(D|R \cap B)$

(ii) $P(D|R \cap B')$

(4)

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast

37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.

(2)