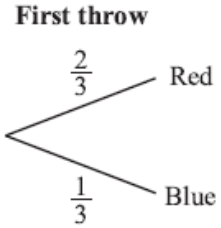
Exam Questions – Set 4

1.

A game is played with a fair, six-sided die which has 4 red faces and 2 blue faces. One turn consists of throwing the die repeatedly until a blue face is on top or until the die has been thrown 4 times.

(i) In the answer book, complete the probability tree diagram for one turn.



[2]

(ii) Find the probability that in one particular turn the die is thrown 4 times. **[2]**

(iii) Adnan and Beryl each have one turn. Find the probability that Adnan throws the die more times than Beryl. **[4]**

2.

Froox sweets are packed into tubes of 10 sweets, chosen at random. 25% of Froox sweets are yellow.

(i) Find the probability that in a randomly selected tube of Froox sweets there are

(a) exactly 3 yellow sweets, **[3]**

(b) at least 3 yellow sweets. **[2]**

(ii) Find the probability that in a box containing 6 tubes of Froox sweets, there is at least 1 tube that contains at least 3 yellow sweets. **[3]**

3.

Each time Ben attempts to complete a crossword in his daily newspaper, the probability that he succeeds is $\frac{2}{3}$. The random variable X denotes the number of times that Ben succeeds in 9 attempts.

(i) Find

(a) $P(X = 6)$, **[3]**

(b) $P(X < 6)$, **[1]**

Ben notes three values, X_1, X_2 and X_3, of X.

(ii) State the total number of attempts to complete a crossword that are needed to obtain three values of X. Hence find $P(X_1 + X_2 + X_3 = 18)$. **[4]**

4.

It is known that on average 85% of seeds of a particular variety of tomato will germinate. Ramesh selects 15 of these seeds at random and sows them.

(i) (*A*) Find the probability that exactly 12 germinate. **[3]**

(*B*) Find the probability that fewer than 12 germinate. **[2]**

The following year Ramesh finds that he still has many seeds left. Because the seeds are now one year old, he suspects that the germination rate will be lower. He conducts a trial by randomly selecting n of these seeds and sowing them. He then carries out a hypothesis test at the 1% significance level to investigate whether he is correct.

(ii) Write down suitable null and alternative hypotheses for the test. Give a reason for your choice of alternative hypothesis. **[4]**

(iii) In a trial with $n = 20$, Ramesh finds that 13 seeds germinate. Carry out the test. **[4]**

(iv) Suppose instead that Ramesh conducts the trial with $n = 50$, and finds that 33 seeds germinate. Given that the critical value for the test in this case is 35, complete the test. **[3]**

(v) If n is small, there is no point in carrying out the test at the 1% significance level, as the null hypothesis cannot be rejected however many seeds germinate. Find the least value of n for which the null hypothesis can be rejected, quoting appropriate probabilities to justify your answer. **[3]**

5.

A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14 781 049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

(i) Explain why Dipak omitted the number 810 from his list. **[1]**

(ii) Explain why Dipak's sample is not random. **[1]**

The mean height of all trees of this species is known to be 4.2 m. Dipak wishes to test whether the mean height of trees in the forest is less than 4.2 m. He now uses a correct method to choose a random sample of 50 trees and finds that their mean height is 4.0 m. It is given that the standard deviation of trees in the forest is 0.8 m.

(iii) Carry out the test at the 2% significance level. **[7]**

6.

The discrete random variable X takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$\mathrm{P}(X=x)=\begin{cases} a & x=1, \\ \frac{1}{2}\mathrm{P}(X=x-1) & x=2,3,4,5, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a=\frac{16}{31}$. **[2]**

The discrete probability distribution for X is given in the table.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

(ii) Find the probability that X is odd. **[1]**

Two independent values of X are chosen, and their sum S is found.

(iii) Find the probability that S is odd. **[2]**

(iv) Find the probability that S is greater than 8, given that S is odd. **[3]**

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable Y defined as follows.

$$\mathrm{P}(Y=y+1)=\frac{1}{2}\mathrm{P}(Y=y) \quad \text{for all positive integers } y.$$

(v) Find $\mathrm{P}(Y=1)$. **[2]**

(vi) Give a reason why one of the variables, X or Y, might be more appropriate as a model for the number of attempts that Sheila needs to start her car. **[1]**