Exam Questions – Set 4 - Answers

1.

(i)		Correct structure with no extra branches Probs and R and B all correct	B1 B1dep **[2]**	Allow extra branches with correct 0 & 1 Ignore other probs	ignore probs and R & B
(ii)		$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ $= \frac{8}{27}$ or 0.296 (3 sf)	M1 A1 **[2]**	or $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ NOT $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ No ft from tree for A1	ft their tree, eg "without replacement" gives $\frac{2}{3} \times \frac{3}{5} \times \frac{2}{4}$ $(=\frac{1}{5})$ M1A0
(iii)		There are basically 6 cases, as in LH column. Some group these into 3 cases, as in middle column. Others use 9 cases - treat these as grouped into the 6 cases below and mark accordingly. Must decide which case a candidates is using, and use the corresponding scheme. NB. Listing Adnan and Beryl separately gains no marks. They must be combined into cases. For example $(\frac{2}{3})^3 \times \frac{1}{3} = \frac{8}{81}$ is correct for P(Adnan 4 throws and Beryl 1throw), but it could also come from P(Adnan gets RRRB) which scores no marks by itself. If correct 6 (or 3) cases, or equiv, are given, but extra cases also given, award 1st M1, and possibly 2nd M1, but no more. No ft from tree, but if clearly state cases may score 1st M1 only.			
		All six cases seen or implied: 2&1; 3&2, 3&1; 4&3, 4&2, 4&1 or 2&1; 3 & (< 3); 4& (< 4)	M1	All three cases soi: R & B; RR & RB; RRR & RRB M1 or R & B; RR & RB; RRRR & RRB; RRRB & RRB ie 3 cases: (≥2 & 1) (≥3 & 2) (4 & 3)	All four cases soi: B&B; RB & RB; RRB & RRB; RRRX & RRRX ie 1&1 or 2&2 or 3&3 or 4&4 M1
		$P(2\&1) = \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}$ or $\frac{2}{27}$		$P(R \& B) = \frac{2}{3} \times \frac{1}{3}$ or $\frac{2}{9}$ M1	$(\frac{1}{3})^2 + (\frac{2}{3} \times \frac{1}{3})^2 + (\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3})^2 + (\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3})^2$ or $\frac{1}{9} + \frac{4}{81} + \frac{16}{729} + \frac{64}{729}$ or $\frac{197}{729}$ all correct M1
		$P(3\&2) = (\frac{2}{3})^2 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$ or $\frac{8}{243}$		**NB Must be clearly part of 3-case method**	$\frac{1}{2}(1 - `\frac{197}{729}')$ M1
		$P(3\&1) = (\frac{2}{3})^2 \times \frac{1}{3} \times \frac{1}{3}$ or $\frac{4}{81}$		$P(RR \& RB) = (\frac{2}{3})^2 \times \frac{2}{3} \times \frac{1}{3}$ or $\frac{8}{81}$	$\frac{266}{729}$ or 0.365 (3 sf) A1
		$P(4\&3) = (\frac{2}{3})^3 \times (\frac{2}{3})^2 \times \frac{1}{3}$ or $(\frac{2}{3})^4 \times (\frac{2}{3})^2 \times \frac{1}{3} + (\frac{2}{3})^3 \times \frac{1}{3} \times (\frac{2}{3})^2 \times \frac{1}{3}$ or $\frac{32}{729}$		$P(RRR \& RRB) = (\frac{2}{3})^3 \times (\frac{2}{3})^2 \times \frac{1}{3}$ or $\frac{32}{729}$ Both these correct expressions or results and add all 3 cases oe ie completely correct method M1 $\frac{266}{729}$ or 0.365 (3 sf) A1	
		$P(4\&2) = (\frac{2}{3})^3 \times \frac{2}{3} \times \frac{1}{3}$ or $(\frac{2}{3})^4 \times \frac{2}{3} \times \frac{1}{3} + (\frac{2}{3})^3 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$ or $\frac{16}{243}$			
		$P(4\&1) = (\frac{2}{3})^3 \times \frac{1}{3}$ or $(\frac{2}{3})^4 \times \frac{1}{3} + (\frac{2}{3})^3 \times (\frac{1}{3})^2$ or $\frac{8}{81}$		May see other groupings of 6 cases into 3 cases eg 4&(1or2or3) 3&(1or2) 2&1 M1	
		Correct expressions (or results) for 3 of these 6 probs	M1	$\frac{8}{27} \times \frac{19}{27}$ oe or $\frac{152}{729}$ M1	
		Correct expressions (or results) for the other 3 of these 6 probs & no extra cases, and add all 6 cases ie completely correct method	M1	$\frac{4}{27} \times \frac{5}{9} + \frac{2}{9} \times \frac{1}{3}$ oe or $\frac{20}{243} + \frac{2}{27}$ or $\frac{38}{243}$ ie completely correct method M1	NB $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ often seen, usually scores 0. Must be clearly part of 3-case method to score.
		$\frac{266}{729}$ or 0.365 (3 sf)	A1	$\frac{266}{729}$ or 0.365 (3 sf) A1	
		See next page for more	**[4]**		
(iii)	cont	4 COMMON INCORRECT METHODS: All six cases seen or implied: M1 $\frac{2}{27} + \frac{8}{243} + \frac{4}{81} + \frac{32}{2187} + \frac{16}{729} + \frac{8}{243}$ oe M1M0 $= \frac{494}{2187}$ or 0.226 A0		ANOTHER EXAMPLE $\frac{RB}{B}$ $\frac{RRB}{B+RB}$ $\frac{RRRB}{B+RB+RRB}$ $\frac{RRRR}{B+RB+RRB}$ ie 2&1 3&(1or2) 4&(1or2or3) This scores the 1st M1 for all 3 cases soi (The last two "fractions" together make the 3rd case)	ANOTHER INCORRECT METHOD $(\frac{2}{3})^3 \times \frac{1}{3} + (\frac{2}{3})^2 \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3}$ $(\frac{2}{3})^2 \times \frac{1}{3} \times \frac{2}{3} + (\frac{2}{3})^3 \times \frac{1}{3} \times \frac{2}{3}$ $+ (\frac{2}{3})^3 \times \frac{1}{3} \times (\frac{2}{3})^2$ ie attempt 4&1, 3&1, 2&1; 3&2, 4&2; 4&3 Some of these overlap, but 1st, 5th, 6th correct. Overall M1M1M0A0
		$\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + (\frac{2}{3})^2 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$ M0M0M0M0			
		P(2&1) + P(3&2) + P(4&3) $= \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + (\frac{2}{3})^2 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} +$ $(\frac{2}{3})^4 \times (\frac{2}{3})^2 \times \frac{1}{3} + (\frac{2}{3})^3 \times \frac{1}{3} \times (\frac{2}{3})^2 \times \frac{1}{3}$ M0M1M0A0			

2.

(i)	**(a)**	Binomial seen or implied $0.7759 - 0.5256$ or ${}^{10}C_3 \times (1-0.25)^7 \times 0.25^3$ $= 0.250$ (3 sf)	B1 M1 A1 **[3]**	by tables or ${}^{10}C_3$ or ${}^{10}C_7$ Allow 0.25	or by $0.25^a \times 0.75^b$ $(a+b=10)$
(i)	**(b)**	$1 - 0.5256$ or $1 - ((1-0.25)^{10} + 10(1-0.25)^9 \times 0.25$ $+ {}^{10}C_2(1-0.25)^8 \times 0.25^2)$ $= 0.4744$ or 0.474 (3 sf)	M1 A1 **[2]**	or $P(X = 3,4,5,6,7,8,9,10)$ all correct terms Allow ${}^{10}C_8$ instead of ${}^{10}C_2$	NOT $1 - 0.7759$ ($P(X>3)$ from table)
(ii)		0.4744 or 0.474) or 0.5256 or 0.526 seen $1 - (1 - \text{“}0.4744\text{”})^6$ oe $= 0.979$ (3 sf)	M1 M1 A1f **[3]**	Their (i)(b) seen, or result of 1-(i)(b) seen or $P(X = 1,2,3,4,5,6)$ all correct terms seen ft from **(i)(b)**	eg B(6, 0.474) or $P(X \geq 3) = 0.474$

3.

(i)	**(a)**	Binomial seen or implied $0.6228 - 0.3497$ $= 0.273$ (3 sf)	M1 M1 A1 **[3]**	by use of table or ${}^9C_6$ or $(\frac{2}{3})^p(\frac{1}{3})^q$ $(p+q=9)$ ${}^9C_6(\frac{1}{3})^3(\frac{2}{3})^6$ $\frac{1792}{6561}$	Eg 0.6228 seen
(i)	**(b)**	0.3497 or 0.350 (3 sf)	B1 **[1]**	NB 0.3498 (from 0.6228 - 0.273) rounds to 0.350 so B1	
(ii)		27 seen B(27, $\frac{2}{3}$) seen or implied ${}^{27}C_{18}(\frac{1}{3})^9(\frac{2}{3})^{18}$ $= 0.161$ (3 sf)	B1 M1 M1 A1 **[4]**	not necessarily in a statement or attempt eg $P(X_1=1) \times P(X_2=8) \times P(X_3=9)$, $P(X_1=2) \times P(X_2=7) \times P(X_3=9)$, $P(X_1=3) \times P(X_2=6) \times P(X_3=9)$, etc $\geq$3 sets with $X_1+X_2+X_3=18$ (not nec'y added) M1	NB $P(X_1=6) \times P(X_2=6) \times P(X_3=6)$ $= 0.273^3 = 0.0203$ M0M0A0 $\frac{55}{729}$ $(= 0.0754)$ M0M0A0

4.

(i)	*(A)*	$X \sim B(15, 0.85)$ P(exactly 12 germinate) $= \binom{20}{4} \times 0.25^4 \times 0.75^{16}$ $= 0.2184$ **OR** from tables: $0.3958 - 0.1773$ $= 0.2185$	M1 M1 A1 **OR** M2 A1 **[3]**	For $0.85^{12} \times 0.15^3$ For $\binom{15}{12} \times p^{12} \times q^3$ CAO For $0.3958 - 0.1773$ CAO
(i)	*(B)*	$P(X<12) = P(X \leq 11) = 0.1773$	M1 A1	For $P(X \leq 11)$ or $P(\leq 11)$ (With no extras) CAO (as final answer) May see alternative method: $0.3958 - 0.2185 = 0.1773$ 0.3958 - their wrong answer to part (i) scores M1A0

5.

(i)		Only 784 trees and 810 > 784	E1 [1]	2.4	or other similar	
(ii)		eg Each no. not independent of previous no. Each no. is related to the next	E1 [1]	2.3	Allow 2nd digit of each no. is 1st of next Consecutive nos share two digits Ignore all else	or similar correct Digits are re-used
(iii)		H_0: $\mu = 4.2$ H_1: $\mu < 4.2$ where μ is mean height of trees (in the wood) $\overline{X} \sim N(4.2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \leq 4.0$ $P(\overline{X} < 4.0) = 0.038549....$ or 0.039 Compare 0.02 Do not reject H_0 There is insufficient evidence that mean height of these trees in the wood is less than 4.2m.	B1 B1 M1 A1 A1 M1 A1f [7]	1.1 2.5 3.3 3.4 1.1 2.2b 3.5a	Allow other letters except X or $\overline{X}$ One error, eg undefined μ or 2-tail: B0B1 Stated or implied Allow $\overline{X} > 4.0$ or $\overline{X} = 4.0$ BC Allow 0.038 NB 0.038... implies M1A1 dep $P(\overline{X} < 4.0)$ attempted Allow Accept H_0 dep $P(\overline{X} < 4.0)$ attempted In context, not definite; eg "Mean height not less than 4.2m": A0	$\Phi^{-1}(0.98)$ (= 2.054) $4.2 - 2.054 \times \frac{0.8}{\sqrt{50}}$ (= 3.968) comp their 3.968 with 4.0 Can be implied by conclusion

6.

(i)		$a(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1$ soi $a = \frac{16}{31}$	M1 A1 [2]	3.1a 1.1	or $\frac{16}{31}(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1$ oe seen correctly obtained	
(ii)		$P(X = 1, 3 \text{ or } 5) = \frac{21}{31}$ 0r 0.677 or 0.68 (2 sf)	B1 [1]	1.1a		
(iii)		P(sum odd) = P(OE) + P(EO) $= 2 \times \frac{21}{31} \times (1 - \frac{21}{31})$ $= \frac{420}{961}$ or 0.437 or 0.44 (2 sf)	M1 A1 [2]	2.1 1.1	or correct "long" method	Allow without "2 ×"
(iv)		P(Sum > 8 & odd) = P(Sum = 9) = P(4, 5) + P(5, 4) $= \frac{2}{31} \times \frac{1}{31} + \frac{1}{31} \times \frac{2}{31}$ $(= \frac{4}{961})$ $\frac{P(\text{Sum} > 8 \text{ \& odd})}{P(\text{Sum odd})}$ $= '\frac{4}{961}' \div '\frac{420}{961}'$ $= \frac{1}{105}$ or 0.00952 or 0.0095 (2 sf)	M1 M1 A1 [3]	1.1a 2.4 1.1	or $P(> 8) \times P(O \mid > 8)$ $= \frac{5}{961} \times \frac{4}{5}$ Attempt ft their (iii) and their P(Sum > 8 & odd) cao NB $\frac{2}{961} \div \frac{210}{961} = \frac{1}{105}$ M0M1A0	Correct method
(v)		$S_\infty = \frac{p}{1 - 0.5} = 1$ $P(X = 1) = 0.5$	M1 A1 [2]	3.4 3.4	Correct ans, no working M1A1	
(vi)		Eg Y. (Y takes all values, but) X cannot be > 5 Eg X because > 5 is very unlikely	B1 [1]	3.5b	oe, eg Y. It may take more than 5 attempts or "limited no." oe instead of 5	