Mathematical Proof 1

Exercise A

1. Prove that 11 is a prime number.
2. Prove that 83 is a prime number.
3. Prove that all regular polygons with fewer than 7 sides have angles with a whole number of degrees.
4. Prove that no square number less than 100 ends in a 7.
5. Let f(x) be the function that gives the number of factors of x. For example, f(10) = 4 because it has factors 1, 2, 5 and 10.

   Prove that for any single digit positive number $f(n) \leqslant n$.
6. Prove that $n^2 + 2$ is not divisible by 4 for integers between 1 and 5 inclusive.
7. Prove that $n^2 + n$ is always even if $n \in \mathbb{Z}$.
8. Prove that when the square of a whole number is divided by 5, the remainder is either 0, 1 or 4.
9. Prove that $2x^3 + 3x^2 + x$ is always divisible by 6 if x is an integer.
10. The modulus function, $|x|$, is defined as x if x is positive and $-x$ if x is negative so, for example, $|-2| = 2$ and $|5| = 5$. $|0|$ is defined to be 0.

    Prove the triangle inequality: $|a + b| \leqslant |a| + |b|$.

Exercise B

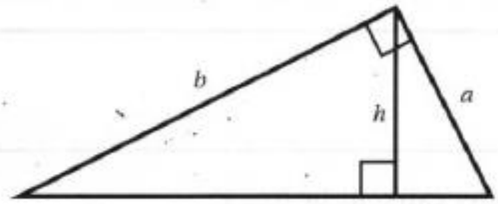
1. Disprove the statement $\sqrt{x^2 + 9} \equiv x + 3$.
2. Use a counter example to prove that $2x \not\equiv 2 \sin x$.
3. Use a counter example to prove that $\sqrt{x^2}$ is not always x.
4. Prove that the product of two prime numbers is not always odd.
5. Prove that the number of factors of a number is not always even.
6. Prove that the sum of two irrational numbers is not always irrational.
7. Use a counter example to disprove the following statement:

$$x < 3 \Rightarrow x^2 < 9$$

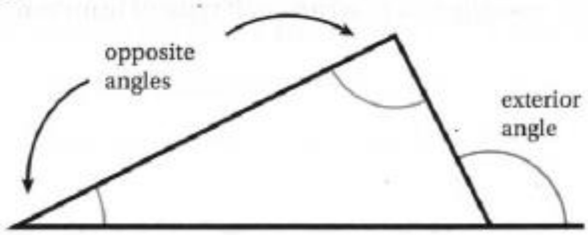
8. A student claims that $n^2 + n + 41$ takes prime values for all positive integers. Use a counter example to disprove this claim.
9. Do two lines that never meet have to be parallel?

Exercise C

1. Prove that if n is odd then n^2 is also odd.
2. Prove that the sum of an even number and an odd number is odd.
3. Prove that the sum of any three consecutive integers is always a multiple of three.
4. Prove that:
   a. the sum of two consecutive multiples of 5 is always odd
   b. the product of two consecutive multiples of 5 is always even.
5. Prove that the height, h, in the following diagram is given by $h = \frac{ab}{\sqrt{a^2+b^2}}$.



6. Prove that the sum of the interior angles of a hexagon is 720°.
7. Prove that if a number leaves a remainder 2 when it is divided by 3 then its square leaves a remainder 1 when divided by 3.
8. a. Expand $(x+2)^2$.
   b. Prove the statement: $y = x^2 + 4x + 10 \Rightarrow y > 0$.
9. Prove that an exterior angle in a triangle is the sum of the two opposite angles.



10. Prove that $n^2 + 3n + 2$ is never prime if n is a positive integer.
11. a. Let n be a four-digit whole number '$abcd$'. Explain why $n = 1000a + 100b + 10c + d$.
    b. Prove that n is divisible by 9 if and only if $a + b + c + d$ is a multiple of 9.
    c. Prove that n is divisible by 11 if and only if $a - b + c - d$ is divisible by 11.
12. By considering $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$, prove that an irrational number raised to an irrational power can be rational.