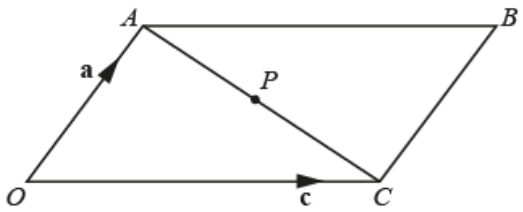
Mixed Exam Questions – Set 1

1.

$OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(i) Find the following in terms of **a** and **c**, simplifying your answers.

(a) $\overrightarrow{AC}$ **[1]**

(b) $\overrightarrow{OP}$ **[2]**

(ii) Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

2.

Points A, B, C and D have position vectors $\mathbf{a} = \begin{pmatrix}1\\2\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}3\\5\end{pmatrix}$, $\mathbf{c} = \begin{pmatrix}7\\4\end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix}4\\k\end{pmatrix}$.

(a) Find the value of k for which D is the midpoint of AC. **[1]**

(b) Find the two values of k for which $\left|\overrightarrow{AD}\right| = \sqrt{13}$. **[3]**

(c) Find one value of k for which the four points form a trapezium. **[2]**

3.

Two particles, A and B, lie at rest on a smooth horizontal plane.

A has position vector $\mathbf{r}_A = (13\mathbf{i} - 22\mathbf{j})$ metres
B has position vector $\mathbf{r}_B = (3\mathbf{i} + 2\mathbf{j})$ metres

(a) Calculate the distance between A and B.

[2 marks]

(b) Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are applied to particle A, where

$\mathbf{F}_1 = (-2\mathbf{i} + 4\mathbf{j})$ newtons

$\mathbf{F}_2 = (6\mathbf{i} - 10\mathbf{j})$ newtons

Given that A remains at rest, explain why $\mathbf{F}_3 = (-4\mathbf{i} + 6\mathbf{j})$ newtons

[1 mark]

(c) A force of $(5\mathbf{i} - 12\mathbf{j})$ newtons, is applied to B, so that B moves, from rest, in a straight line towards A.

B has a mass of 0.8 kg

(c) (i) Show that the acceleration of B towards A is $16.25\,\text{m s}^{-2}$

[2 marks]

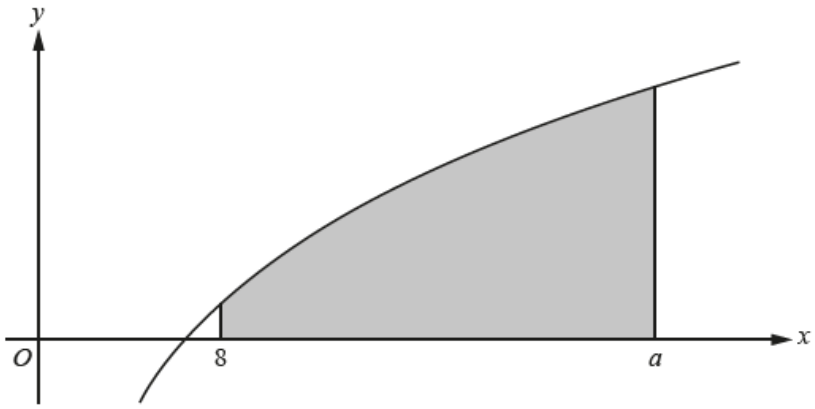
(c) (ii) Hence, find the time taken for B to reach A.

Give your answer to two significant figures.

[2 marks]

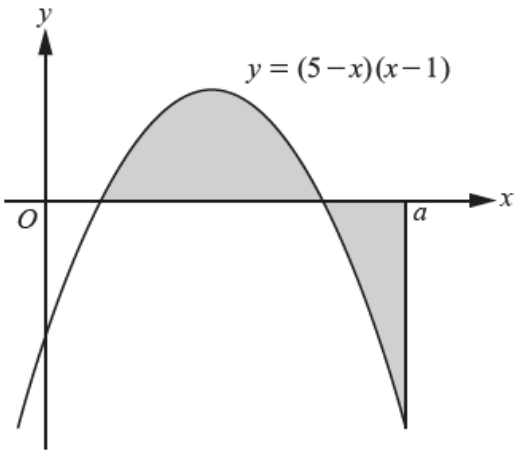
4.

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

5.



The diagram shows part of the curve $y = (5-x)(x-1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a. **[8]**

6.

The equation of a curve is $y = \frac{x^2}{4} + \frac{2}{x} + 1$. A tangent and a normal to the curve are drawn at the point where $x = 2$.

Calculate the area bounded by the tangent, the normal and the x-axis. **[10]**

7.

The probability that Janice sees a kingfisher on any particular day is 0.3. She notes the number, X, of days in a week on which she sees a kingfisher.

(i) State one necessary condition for X to have a binomial distribution. **[1]**

Assume now that X has a binomial distribution.

(ii) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days. **[1]**

Each week Janice notes the number of days on which she sees a kingfisher.

(iii) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks. **[3]**

8.

Nicola, a darts player, is practising hitting the bullseye. She knows from previous experience that she has a probability of 0.3 of hitting the bullseye with each dart.

Nicola throws eight practice darts.

(a) Using a binomial distribution, calculate the probability that she will hit the bullseye three or more times.

[2 marks]

(b) Nicola throws eight practice darts on three different occasions. Calculate the probability that she will hit the bullseye three or more times on all three occasions.

[2 marks]

(c) State two assumptions that are necessary for the distribution you have used in part **(a)** to be valid.

[2 marks]

9.

A mathematical puzzle is published every day in a newspaper.

Over a long period of time Paula is able to solve the puzzle correctly 60% of the time.

(a) For a randomly chosen 14-day period find the probability that:

(a) (i) Paula correctly solves exactly 8 puzzles

[1 mark]

(a) (ii) Paula correctly solves at least 7 but not more than 11 puzzles.

[2 marks]

(b) State one assumption that is necessary for the distribution used in part **(a)** to be valid.

[1 mark]
