Exam Questions – Set 2

1.

The functions f and g are defined for all real values of x by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where a is a positive constant.

(i) State the range of f and the range of g. **[2]**

(ii) State why f has no inverse, and find an expression for $g^{-1}(x)$. **[3]**

(iii) Solve for x the equation $gf(x) = 31a$. **[5]**

2.

(i) Prove that $\dfrac{\sin(\theta - \alpha) + 3\sin\theta + \sin(\theta + \alpha)}{\cos(\theta - \alpha) + 3\cos\theta + \cos(\theta + \alpha)} \equiv \tan\theta$ for all values of α. **[5]**

(ii) Find the exact value of $\dfrac{4\sin 149^\circ + 12\sin 150^\circ + 4\sin 151^\circ}{3\cos 149^\circ + 9\cos 150^\circ + 3\cos 151^\circ}$. **[3]**

(iii) It is given that k is a positive constant. Solve, for $0^\circ < \theta < 60^\circ$ and in terms of k, the equation

$$\frac{\sin(6\theta - 15^\circ) + 3\sin 6\theta + \sin(6\theta + 15^\circ)}{\cos(6\theta - 15^\circ) + 3\cos 6\theta + \cos(6\theta + 15^\circ)} = k.$$ **[4]**

3.

In this question, I denotes the definite integral $\displaystyle\int_2^5 \frac{5 - x}{2 + \sqrt{x - 1}}\,dx$. The value of I is to be found using two different methods.

(i) Show that the substitution $u = \sqrt{x - 1}$ transforms I to $\displaystyle\int_1^2 (4u - 2u^2)\,du$ and hence find the exact value of I. **[5]**

(ii) **(a)** Simplify $\left(2 + \sqrt{x - 1}\right)\left(2 - \sqrt{x - 1}\right)$. **[1]**

(b) By first multiplying the numerator and denominator of $\dfrac{5 - x}{2 + \sqrt{x - 1}}$ by $2 - \sqrt{x - 1}$, find the exact value of I. **[3]**

4.

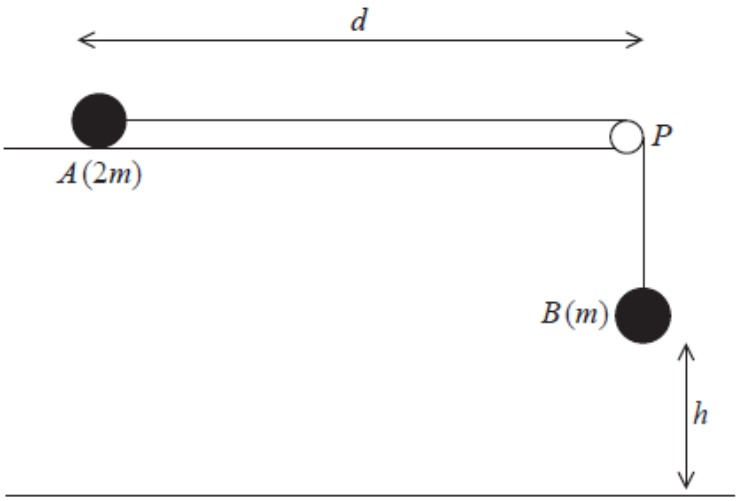

Figure 3

Two particles, A and B, have masses $2m$ and m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a fixed rough horizontal table at a distance d from a small smooth light pulley which is fixed at the edge of the table at the point P. The coefficient of friction between A and the table is μ, where $\mu < \frac{1}{2}$. The string is parallel to the table from A to P and passes over the pulley. Particle B hangs freely at rest vertically below P with the string taut and at a height h, $(h < d)$, above a horizontal floor, as shown in Figure 3. Particle A is released from rest with the string taut and slides along the table.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence show that, until B hits the floor, the acceleration of A is $\frac{g}{3}(1 - 2\mu)$.

(3)

(c) Find, in terms of g, h and μ, the speed of A at the instant when B hits the floor.

(2)

5.

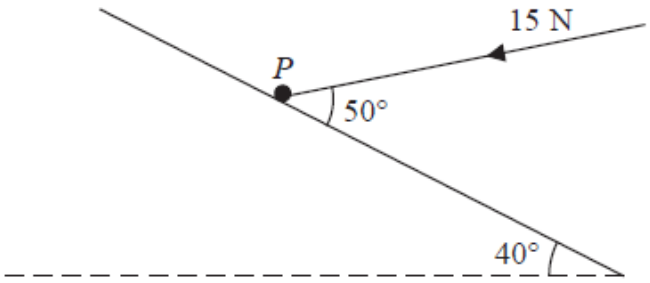

Figure 4

A particle P of mass 2.7 kg lies on a rough plane inclined at 40° to the horizontal. The particle is held in equilibrium by a force of magnitude 15 N acting at an angle of 50° to the plane, as shown in Figure 4. The force acts in a vertical plane containing a line of greatest slope of the plane. The particle is in equilibrium and is on the point of sliding down the plane.

Find

(a) the magnitude of the normal reaction of the plane on P,

(4)

(b) the coefficient of friction between P and the plane.

(5)

The force of magnitude 15 N is removed.

(c) Determine whether P moves, justifying your answer.

(4)

6.

The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a randomly selected adult female has a height greater than 170 cm.

(3)

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm.

(4)

Half of tall adult females have a height greater than h cm.

(c) Find the value of h.

(5)